UCLA Resident Physics Review

Atomic, Nuclear Structure and Radioactivity

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Syllabus

Topics	weeks
1. Atomic, Nuclear Structure and Radioactivity	1
2. Particle Interactions	1
3. Production of radiation and Treatment machines	2
4. Quantification and Measurement of Dose and radiation detection	2
5. Photon Beam Dosimetry	2
6. Electron Dosimetry	1
7. Treatment Planning (3D, ICRU, IMRT, TBI, etc)	1
8. Brachytherapy	1
9. Radiation protection; Physics QA and Patient Safety	1
10. Special modalities in RT (Particle beam therapy)	1
11. Imaging for RT	1
12. Special Topics (Question Review and MU calculation)	1

Reference Textbook:

- The Physics and Technology of Radiation Therapy. Patrick McDermott, Colin Orton.
- http://www.amazon.com/The-Physics-Technology-Radiation-Therapy/dp/1930524447
- http://www.amazon.com/Khans-Lectures-Handbook-Physics-Radiation/dp/1605476811

Guidelines based on ABR recommendations

Atomic, Nuclear Structure & Radioactivity

- Bohr model of the atom, electron transitions, and characteristic radiation
- Nuclear structure, nuclear forces, mass/energy relationships
- Factors affecting nuclear stability
- Nuclear nomenclature
- Modes of radioactive decay
- Decay schemes and properties for therapeutic isotopes
- Mathematics of radioactive decay

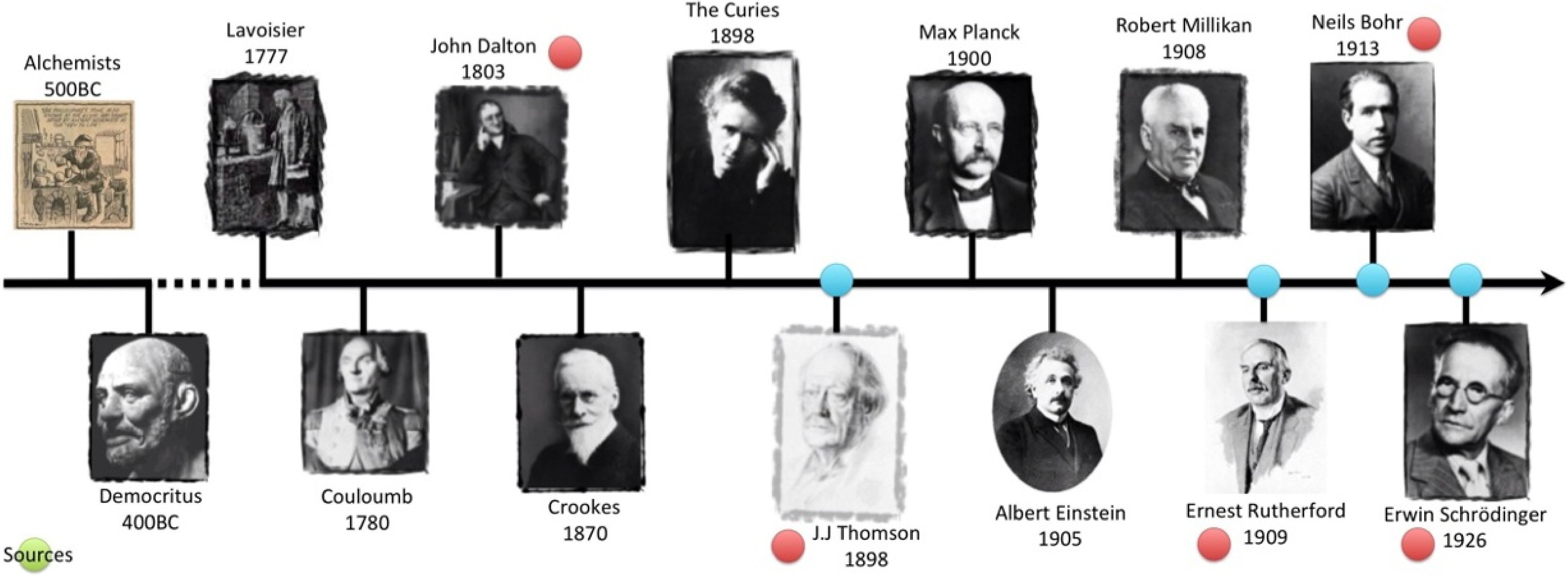
Many inforgraphics in this talk were taken from https://vceasy.files.wordpress.com or http://hyperphysics.phy-astr.gsu.edu

Evolution of Atomic Theory

History of Atomic Models Timeline

New Atomic Model (Picture and Name of Model)

Experiment that lead to the new model (Video)



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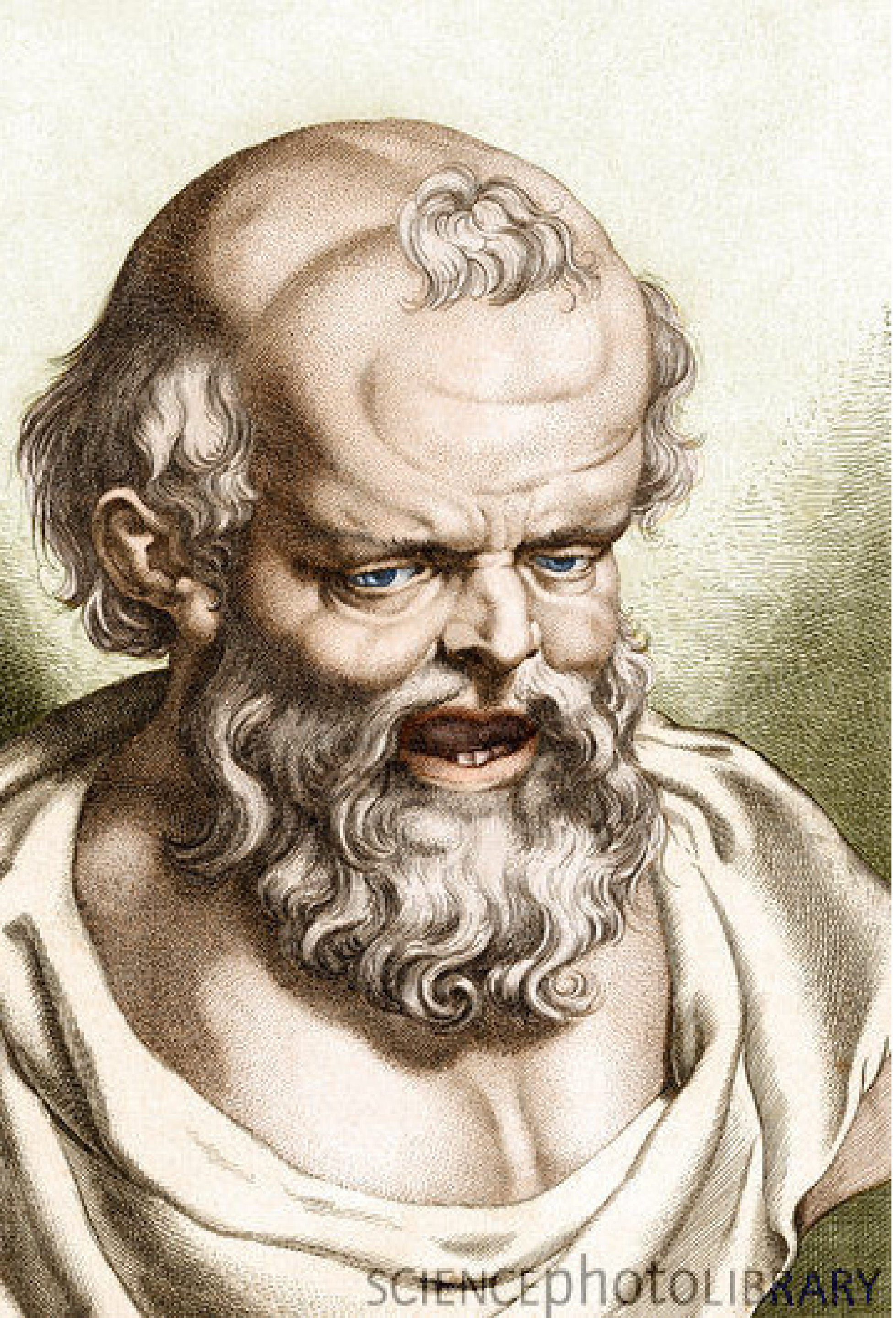
Democritus

- Lived 460-370 BC
- First hypothesized that all matter was made of microscopic spherical elements which he dubbed ‘atoms’
- The word ‘atom’ derives from the Greek ‘atomos’, which means indivisible
- The atomic theory states
  - “The universe is composed of two elements: the atoms and the void in which they exist and move.”



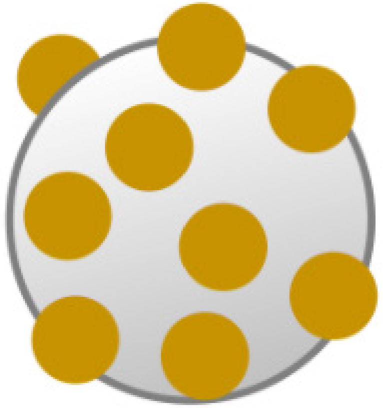
Democritus

- All matter consists of invisible particles called atoms
- Atoms are indestructible
- Atoms are solid but invisible
- Atoms are homogenous
- Atoms differ in size, shape, mass, position, and arrangement

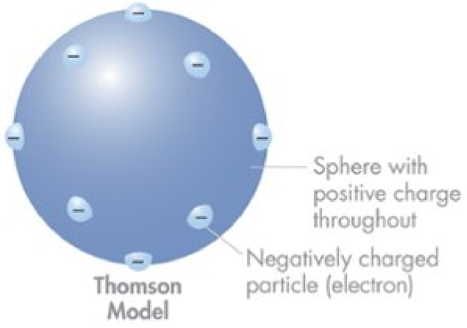
John Dalton

- Lived 1766-1844
- Dalton’s Atomic Theory
  - All matter is made of atoms. Atoms are indivisible and indestructible.
  - All atoms of a given element are identical in mass and properties
  - Compounds are formed by a combination of two or more different kinds of atoms.
  - A chemical reaction is a ***rearrangement*** of atoms.
- Became a theoretical foundation of chemistry

JJ Thomson

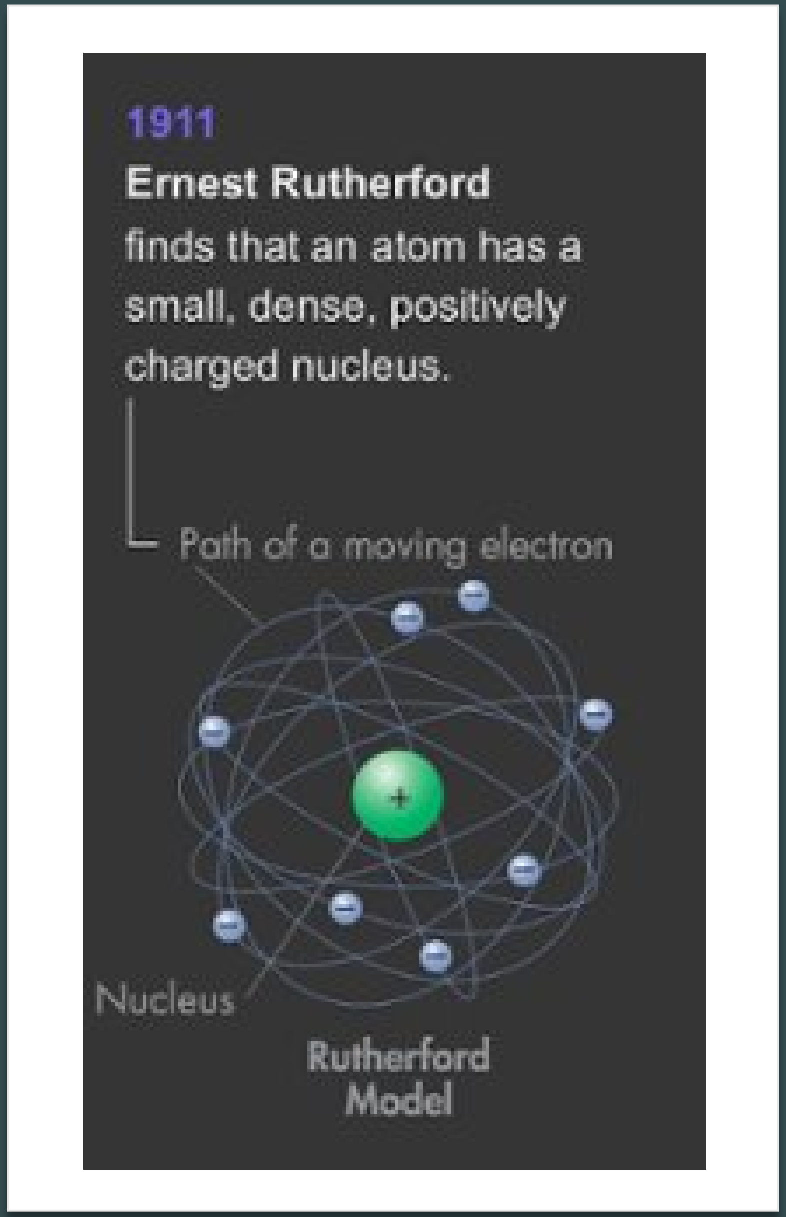


- Lived 1856-1940
- Discovered the electron, which he originally dubbed 'corpuscles'
  - Through experimentation with cathode ray tubes (Crookes Tube)
  - Observed deflection of beam in electric/magnetic fields
- Atoms are divisible
- Proposed a model whereby the negatively charged corpuscles were distributed in a uniform sea of positive charge.
- Plum Pudding Model

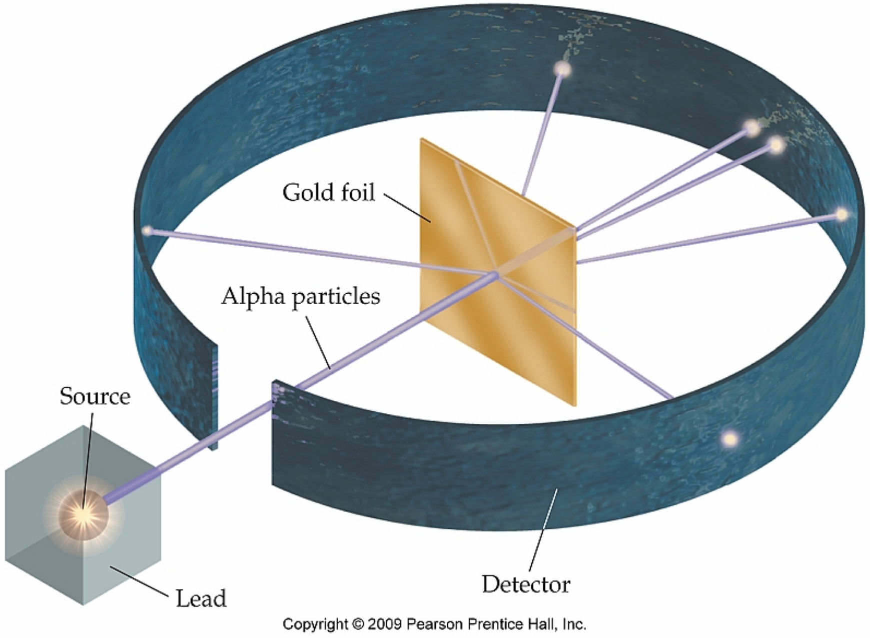


Ernest Rutherford

- Lived 1871-1937
- Through his gold foil experiments, determined that the positive charge of an atom was localized to a small, dense nucleus at its center
  - Also known as a Planetary Atomic Model
- Discovered most of atomic mass was in the nucleus
  - Atoms are mostly empty space
  - Electrons move rapidly in the space around the nucleus
- Proposed the name ‘proton’ for the positively charged particles in the nucleus
- Postulated the existence of neutrons
  - Proven correct by James Chadwick in 1932

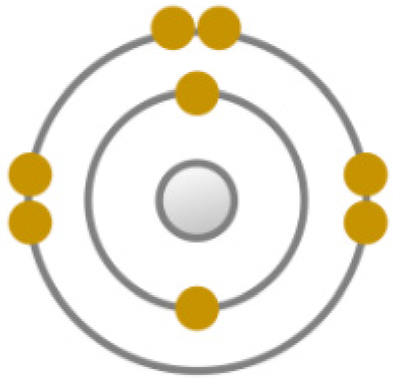


Rutherford Gold Foil Experiment



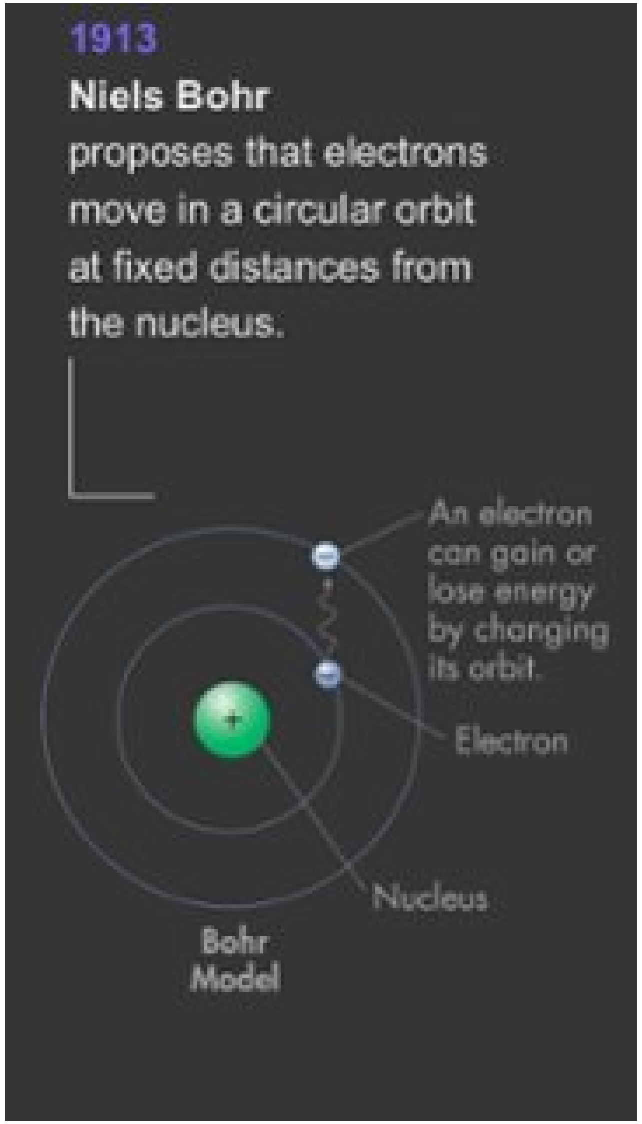
- Shot a beam of alpha particles at a gold foil and observed the deflections
- ~1 in 10^4 particles reflected at angles larger than 90 degrees, indicating that the mass of the atom was concentrated in a small fraction of the total volume
- By measuring the fraction of particles deflected, he was able to estimate the size of the nucleus (radius ~10^{-15} m)
  - Atom radius ~10^{-10} m

Hans Geiger



Neils Bohr

- Lived 1885-1962
- Combined Rutherford's atomic model with Max Planck's idea of the quantized nature of the radiation process
  - E = hv
- First model to successfully deal with 1 electron structures
- Most of the mathematics of radioactive decay are based off Bohr's model
  - Based on Classical Mechanics

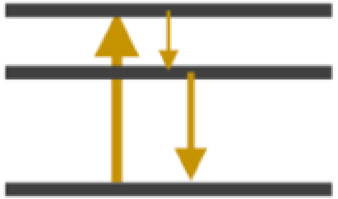


Bohr’s Postulates

1) Electrons revolve about the Rutherford nucleus in well-defined, allowed orbits
   - Planetary-like motion
2) While in orbit, the electron does not lose any energy despite being constantly accelerated
   - No energy loss while electron is in allowed orbit
3) The angular momentum of the electron in an allowed orbit is quantized
   - Quantization of angular momentum
4) An atom emits radiation only when an electron makes a transition from one orbit to another
   - Energy emission during orbital transitions

Limitations of Bohr’s Model

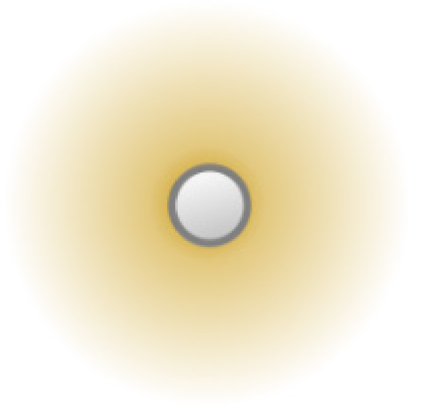
Why do electrons move in circular orbits?



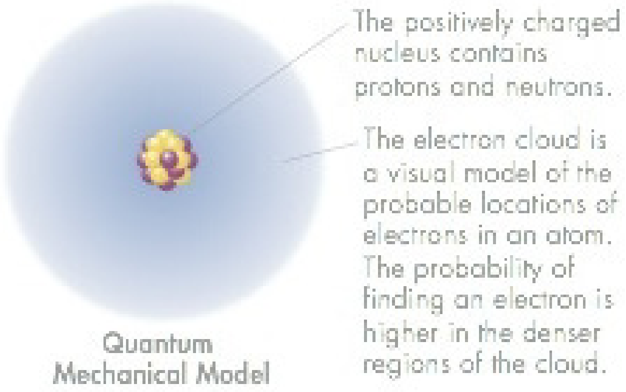
Why do shells have particular energies?

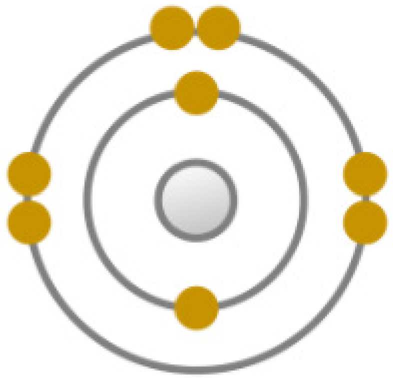
Why do shells allow more than 8 electrons?

Erwin Schrödinger



- Lived 1887-1961
- Electron Cloud Model or Quantum Mechanical Model
- Electrons have wave properties
- Locations around the nucleus are probabilistic
- More modern advancements:
  - Protons/Neutrons are made of quarks
  - Quarks are made of vibrating strings

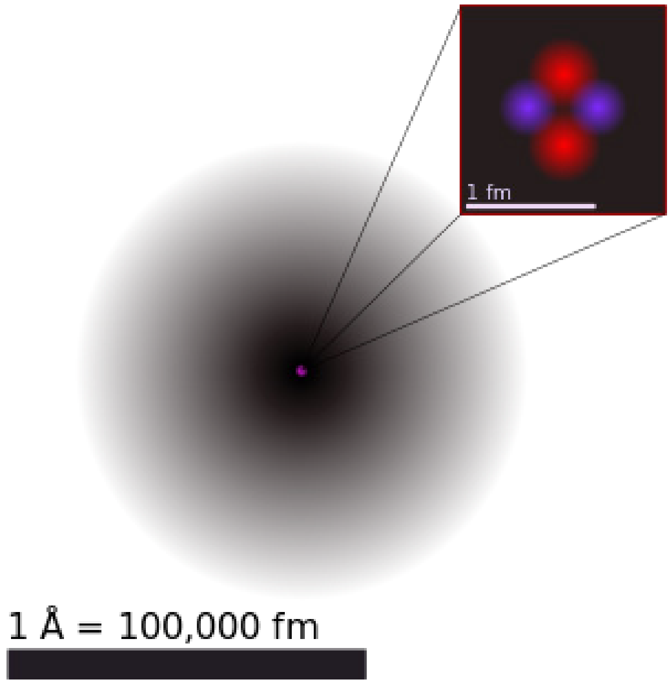




Bohr Model of the Atom

Atomic Structure

- Vast majority of mass is concentrated in the nucleus: >= 99.95%.
  - Nucleus radius ~ 1 femtometer (10^{-15} m)
  - Atomic radius ~ 1 Angstrom (10^{-10} m)
- Neutron mass: 1.675×10^{-27} kg (about 1u).
- Proton mass: 1.673×10^{-27} kg (about 1u).
- Electron mass: 9.11×10^{-31} kg (about 0.0005u).



Atomic Mass Units (Daltons)

The **Dalton** (or atomic mass unit) is a unit of mass defined as 1/12 weight of carbon-12 in its nuclear and electronic ground state.

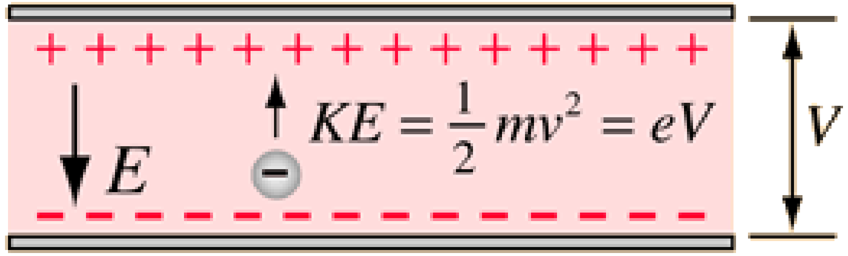
1 Da = 1 u = 1/12 m(^{12}C)

The dalton is equal to 1.660538921(73) × 10^{-27} kilograms (1 Da = 1.660538921(73) × 10^{-27} kg), the base unit of mass in the International System of Units (SI). It is also equal to 1,822.88839 electron rest mass (me).

The energy equivalent of 1 dalton:
1 Da = 1 amu = 931.494061(21) × 10^6 eV/c^2 = 931.494061(21) MeV/c^2 = 0.931494061(21) GeV/c^2

Electron Volts

- The energy given to an electron when accelerating it through 1 Volt of electric potential difference



Work done on electron W:

$W = qV = (1.6x10^{-19}C)(1\frac{J}{C})$

$W = 1$ electron volt $= 1.6x10^{-19}J$

e = electron charge $= 1.6x10^{-19}C$

V = voltage E = electric field

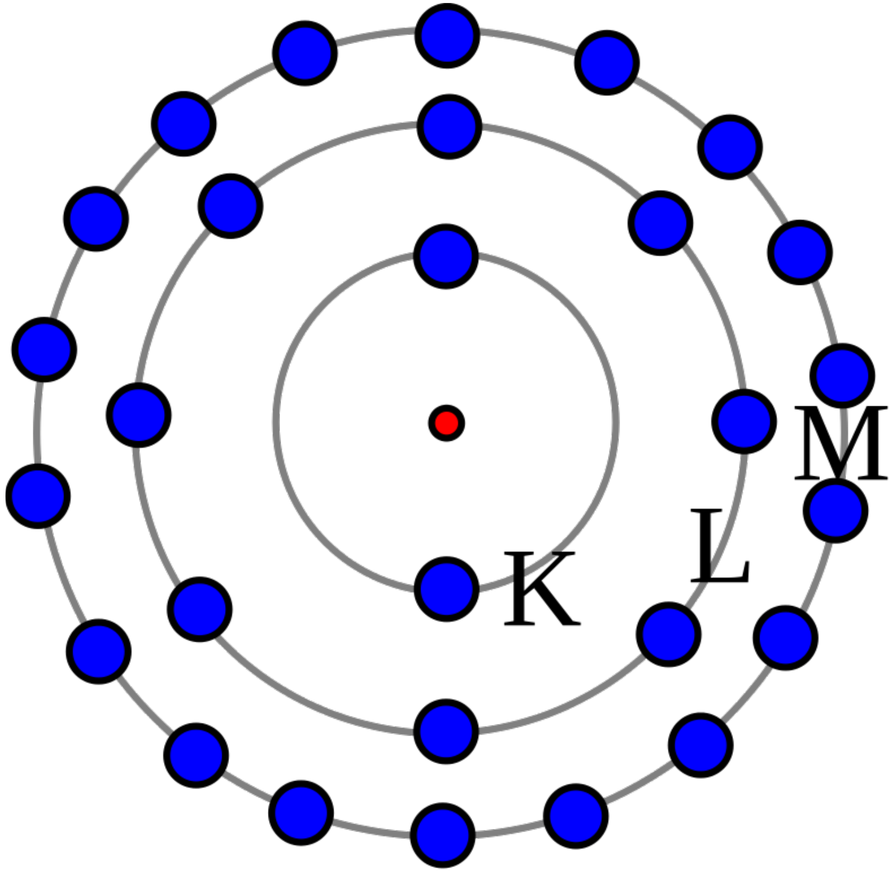
Electron-volts **1 eV = 1.6×10^{-19} J**

Electron Orbitals

- Bound electrons are allowed only in certain orbits, in which the angular momentum of the electron is an integer multiple of Planck's constant, h.

$$L = mvr = \frac{nh}{2\pi}$$

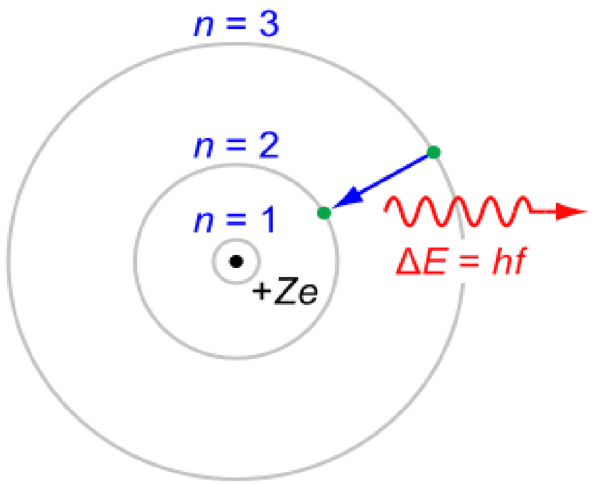
- The closest shell to the nucleus is called the "K shell", followed by the "L shell", then the "M shell", and so on farther and farther from the nucleus.
- Only a fixed number of electrons are allowed in each shell ($2n^2$, where n indexes K,L,M…)



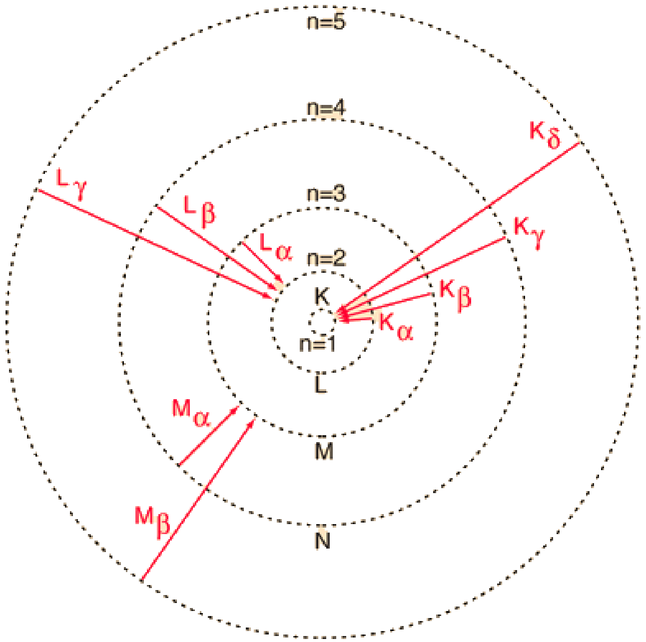
Planck's Constant, h = 6.626×10^{-34} J·s = 4.136×10^{-15} eV·s

Electron Orbitals

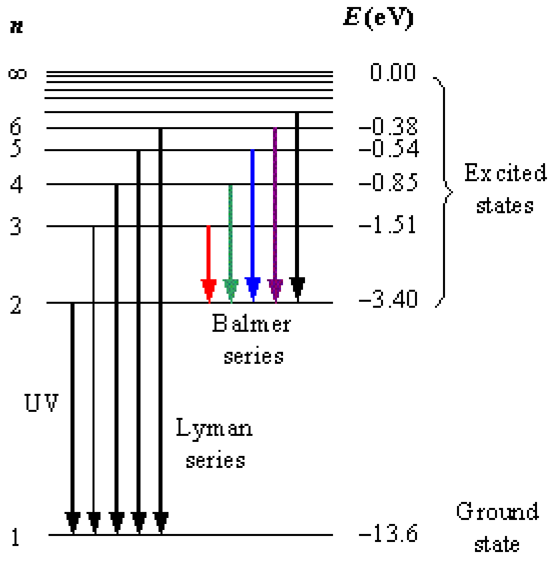
- An electron jump between orbits is accompanied by an emitted or absorbed amount of electromagnetic energy (hv) as photons



- These photons are called Characteristic X-Rays



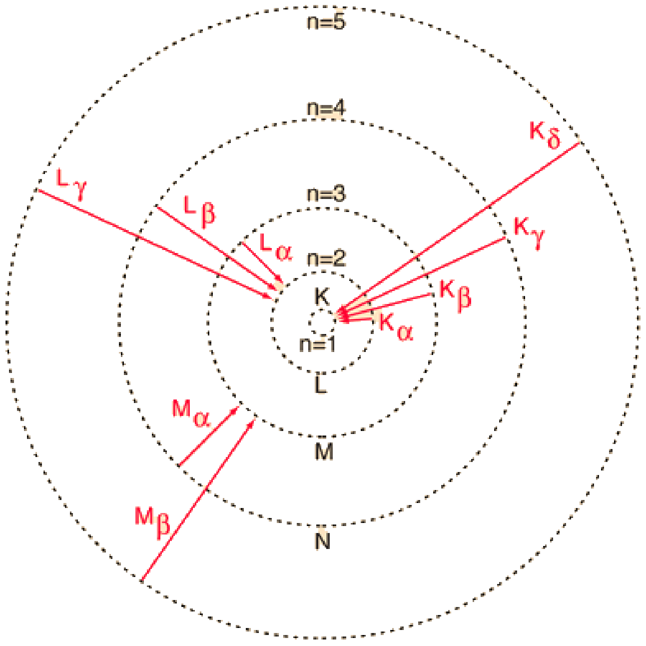
Energy States of Hydrogen



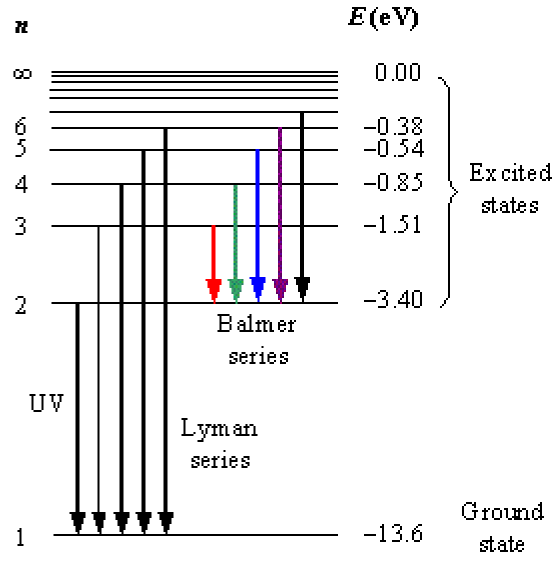
Electron Binding Energy

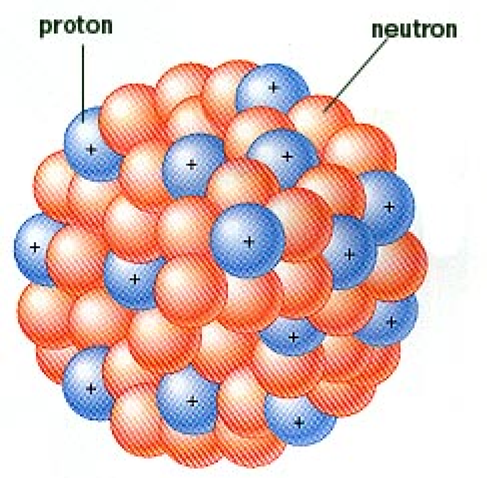
- This is the energy required to remove an electron completely from the atom.
- By convention, binding energies are negative with increasing magnitude for electrons in shells closer to the nucleus.
- For an electron to be removed from its shell, the energy transferred to it (from a photon or particle) must be ≥ its binding energy.
- Energy involved in transitioning from K shell to L shell for a given atom is:

$$\Delta E = E_{b,K} - E_{b,L}$$



Energy States of Hydrogen

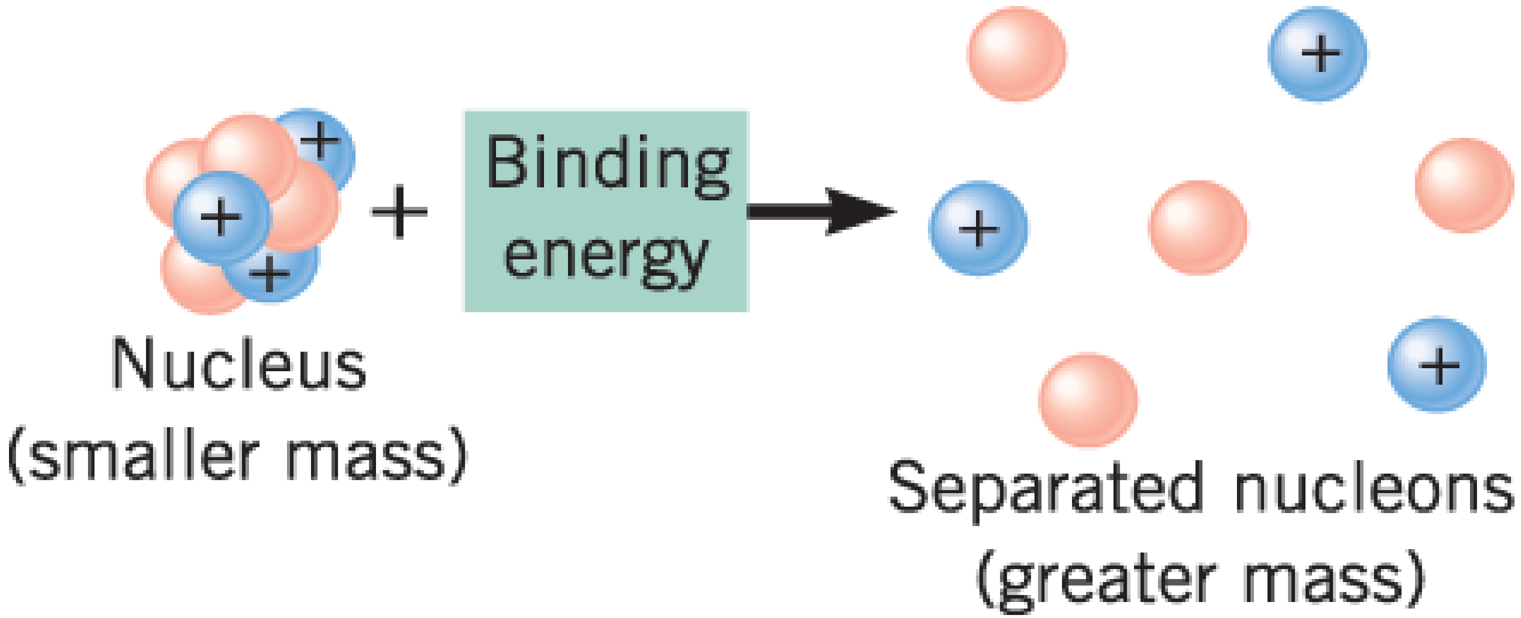




Nuclear Structure & Stability

Binding Energy of the Nucleus

- The binding energy of the nucleus is equal to the mass defect
- It is the energy which is required to split the nucleus into individual nucleons



Equivalence of Mass & Energy

Energy, J

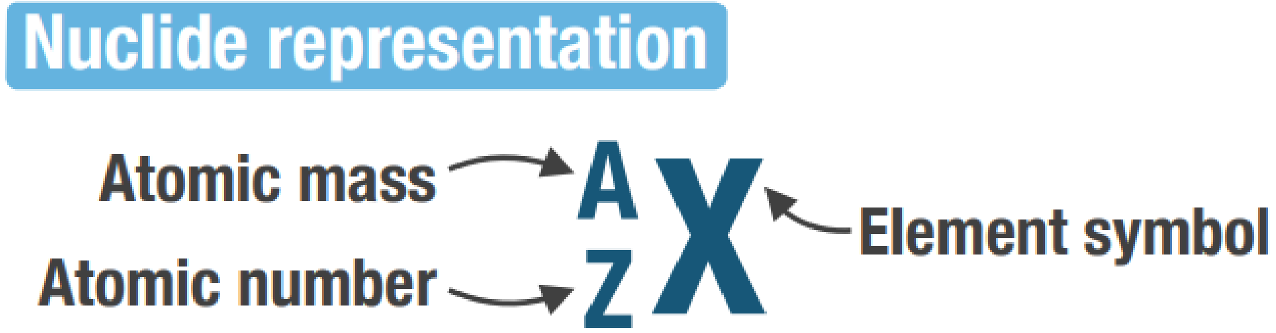
mass, kg

$$E = mc^2$$

velocity of light, m s^{-1}

Atomic Nomenclature

- Atomic mass is the number of protons and neutrons in the atom
- Atomic number is the number of protons



Atomic Nomenclature

isotoPe

- Same number of Protons (Z), Different A

isotoNe

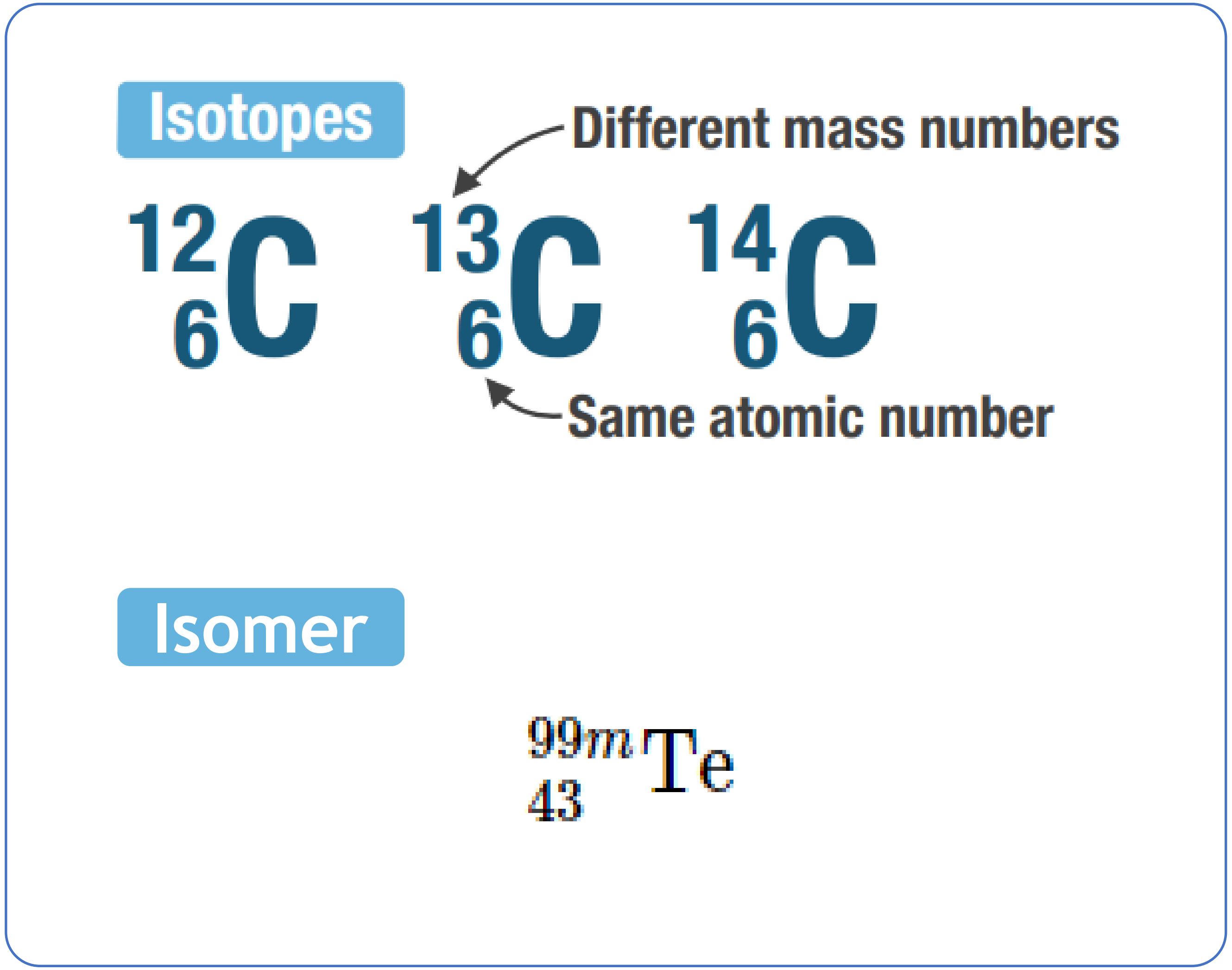
- Different elements with the same number of Neutrons

isoMer

- Nucleus in a Metastable energetic state

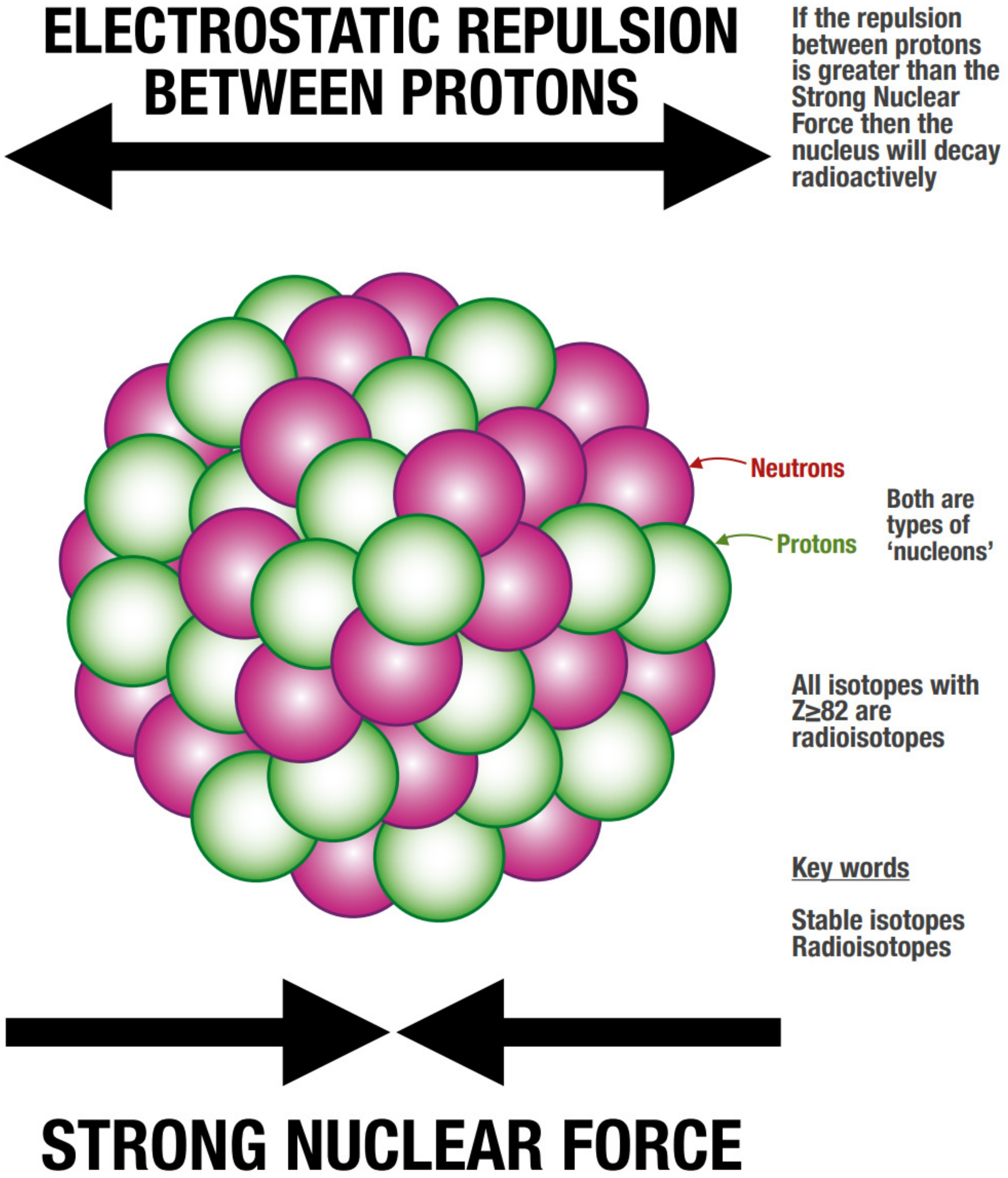
isobAr

- Different elements with the same Atomic mass



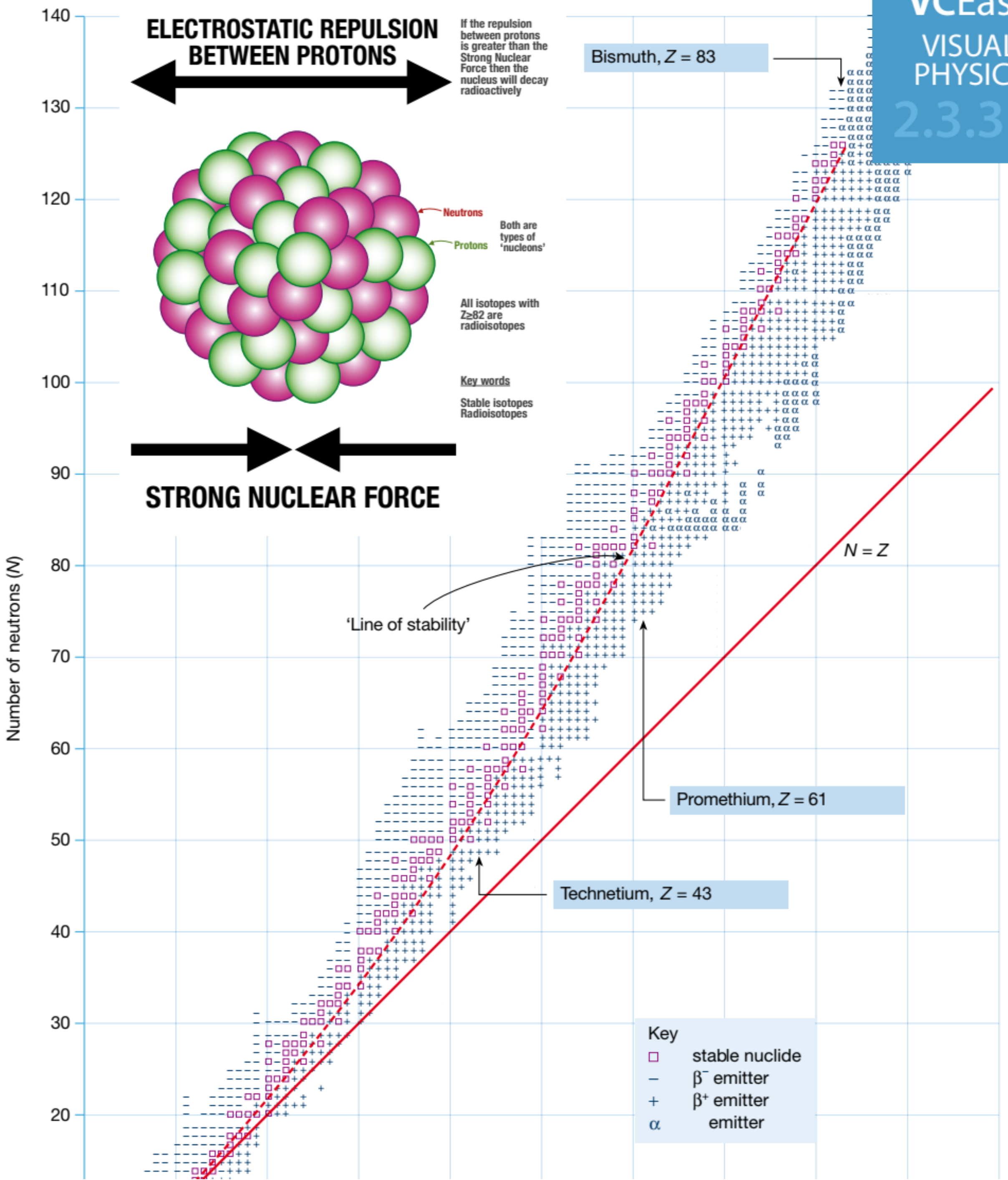
Nuclear Forces

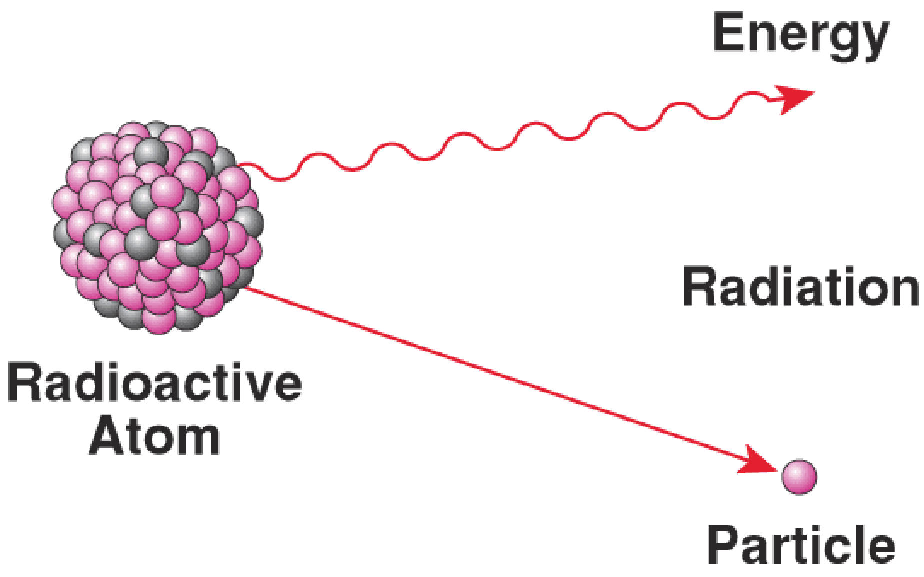
- In general, stable nuclides have an equal number of protons and neutrons
- However, as the Z increases the number of neutrons increases slightly
- Protons repel each other and neutrons can be thought of as glue which holds the nucleus together (you need more glue as the repulsive force grows)



Line of Stability

- Radioactive decay is the process in which an unstable atom releases matter and/or energy during a transition to a more stable form
- Atoms that are unstable are also known as radioactive atoms, or radionuclides
- Nuclides attempt to return to the line of stability through an appropriate mode of radioactive decay

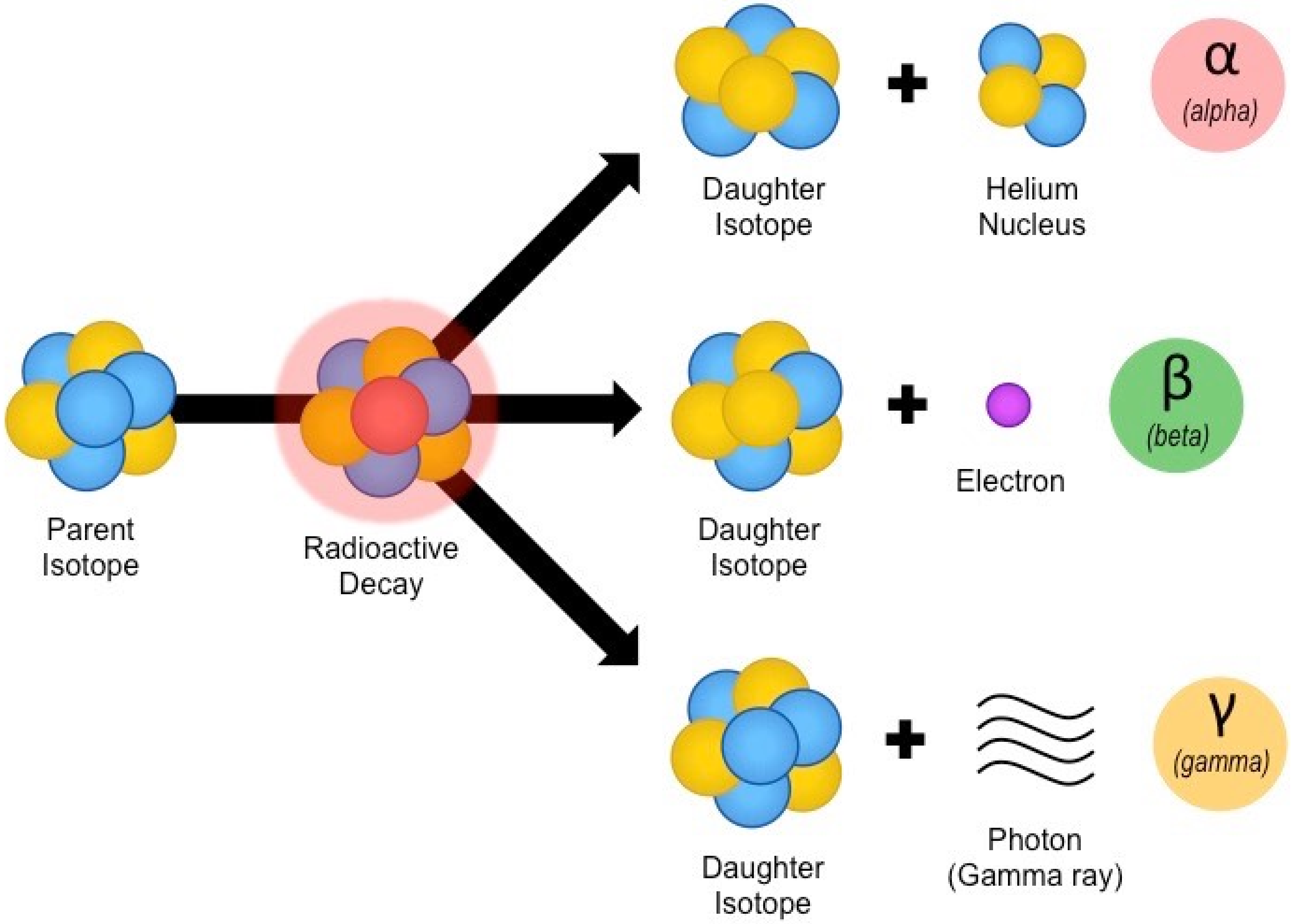




Radioactive Decay

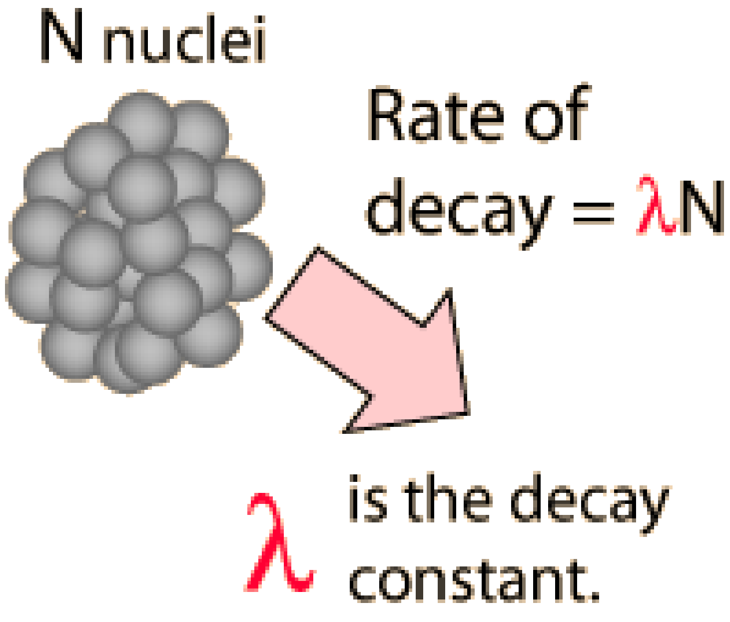
Radioactive Decay

- Radionuclides may release subatomic particles and energy, or capture an orbital electron into the nucleus and release energy
- The original, radioactive atom is known as the parent. The new nucleus (after decay) is known as the daughter.



Radioactive Decay

- Decay is stochastic
- The exact moment of an individual atomic decay cannot be predicted, the probability of decay during a given time period can be measured (based on observations from a large number of atoms)
- This quantity is known as the decay constant (λ)
- The decay constant is expressed in units of probability per unit time
- The theory of radioactive decay is built around the understanding that the number of atoms disintegrating per time is proportional to the number of atoms in the population



$$\Delta N / \Delta t = -\lambda N$$

Radioactive Decay

- If delta is small, this expression becomes differentiable and can be solved to provide the following solution for the number of radioactive atoms remaining after time, *t*

$$\frac{dN}{dt} = -\lambda N$$

or, by rearranging,

$$\frac{dN}{N} = -\lambda dt.$$

Integrating, we have

$$\ln N = -\lambda t + C$$

where C is the constant of integration,

$$N(t) = e^{C} e^{-\lambda t} = N_0 e^{-\lambda t}$$

Activity

- Activity is proportional to the number of atoms; then activity *A*, or the rate of change of the radioactive sample, is:

$$A = -\frac{dN}{dt} = \lambda N$$

Unit	Abbreviation	Multiple	Type of Unit	Number of Disintegrations per Second (dps)
curie	Ci	1 Ci	Traditional	37,000,000,000
millicurie	mCi	10^{-3} Ci	Traditional	37,000,000
microcurie	:Ci	10^{-6} Ci	Traditional	37,000
nanocurie	nCi	10^{-9} Ci	Traditional	37
becquerel	Bq	1 Bq	SI	1
kilobecquerel	kBq	10^{3} Bq	SI	1000
megabecquerel	mBq	10^{6} Bq	SI	1,000,000
gigabecquerel	gBq	10^{9} Bq	SI	1,000,000,000

LDR = mCi
HDR = ~10Ci
ViewRay = 13 KCi

Half-Life

- Rate of decay can also be described by the half-life of the atom
- The time required for 50% of the parent atoms to undergo radioactive decay

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

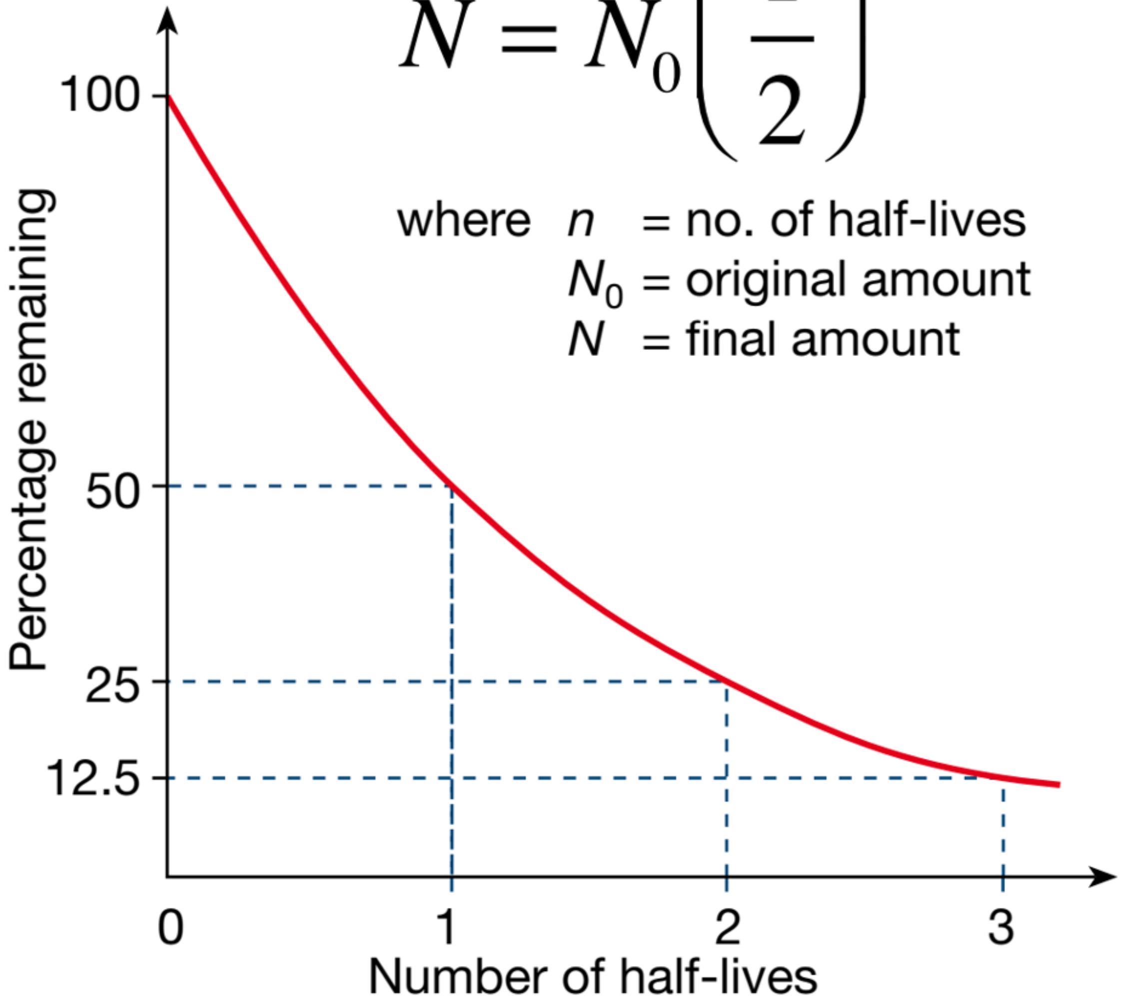
$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{0.693}{t_{1/2}}$$

Half-life

Half-life is the time taken for half the radioactive nuclei to decay spontaneously

$$N = N_0\left(\frac{1}{2}\right)^n$$

where n = no. of half-lives
N_0 = original amount
N = final amount



Becquerels

Decay is measured in Becquerels (Bq)
1 Bq = 1 disintegration per second

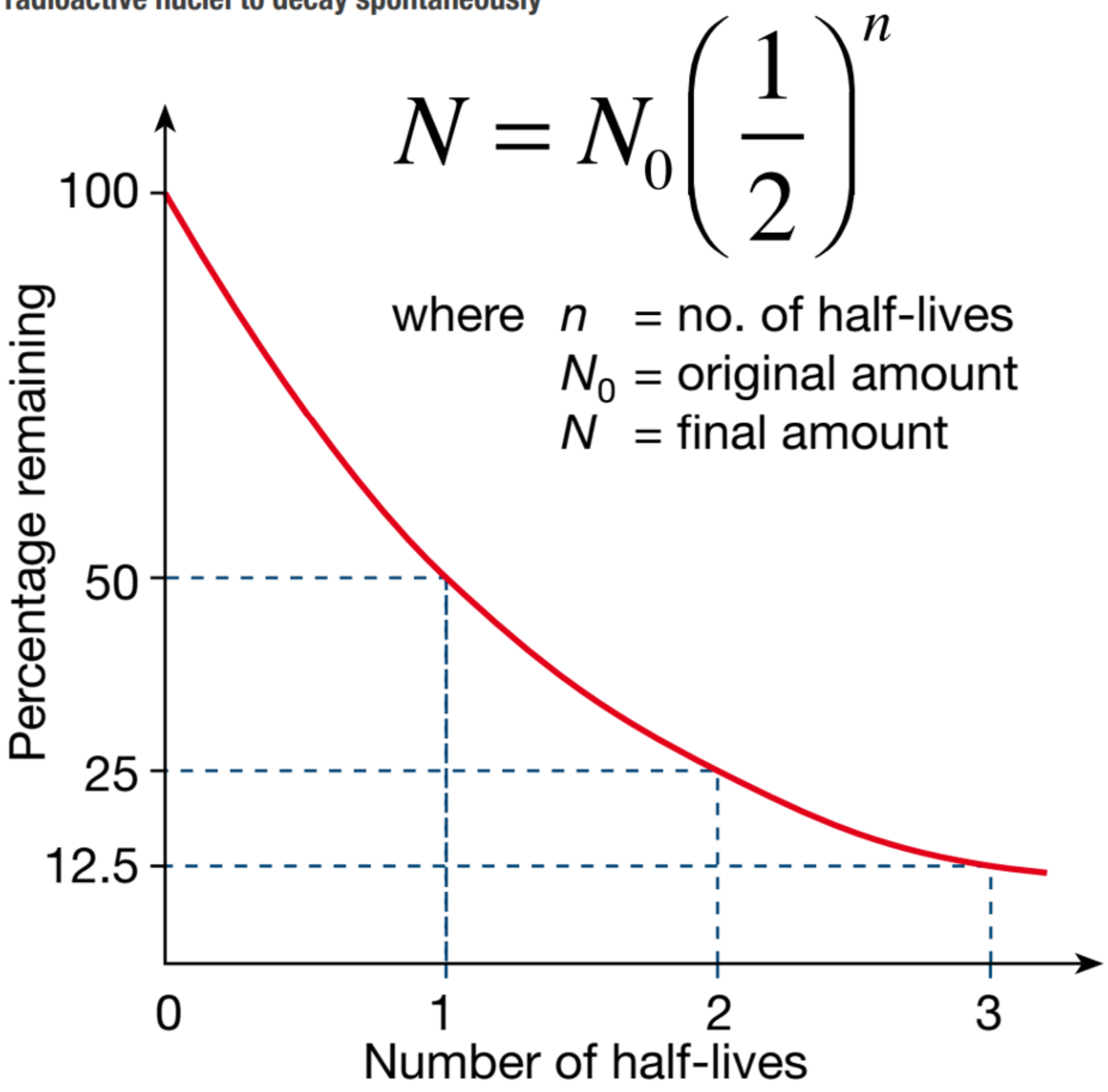
Mean-Life

- The average life-span of one of the members of a radioactive sample
- This term exists with perspective to statistics only as no one member of the radioactive sample can be predictable in its decay activities

$$(\tau = \frac{1}{\lambda} = 1.443 * T_{1/2})$$

Half-life

Half-life is the time taken for half the radioactive nuclei to decay spontaneously



Becquerels

Decay is measured in Becquerels (Bq)

1 Bq = 1 disintegration per second

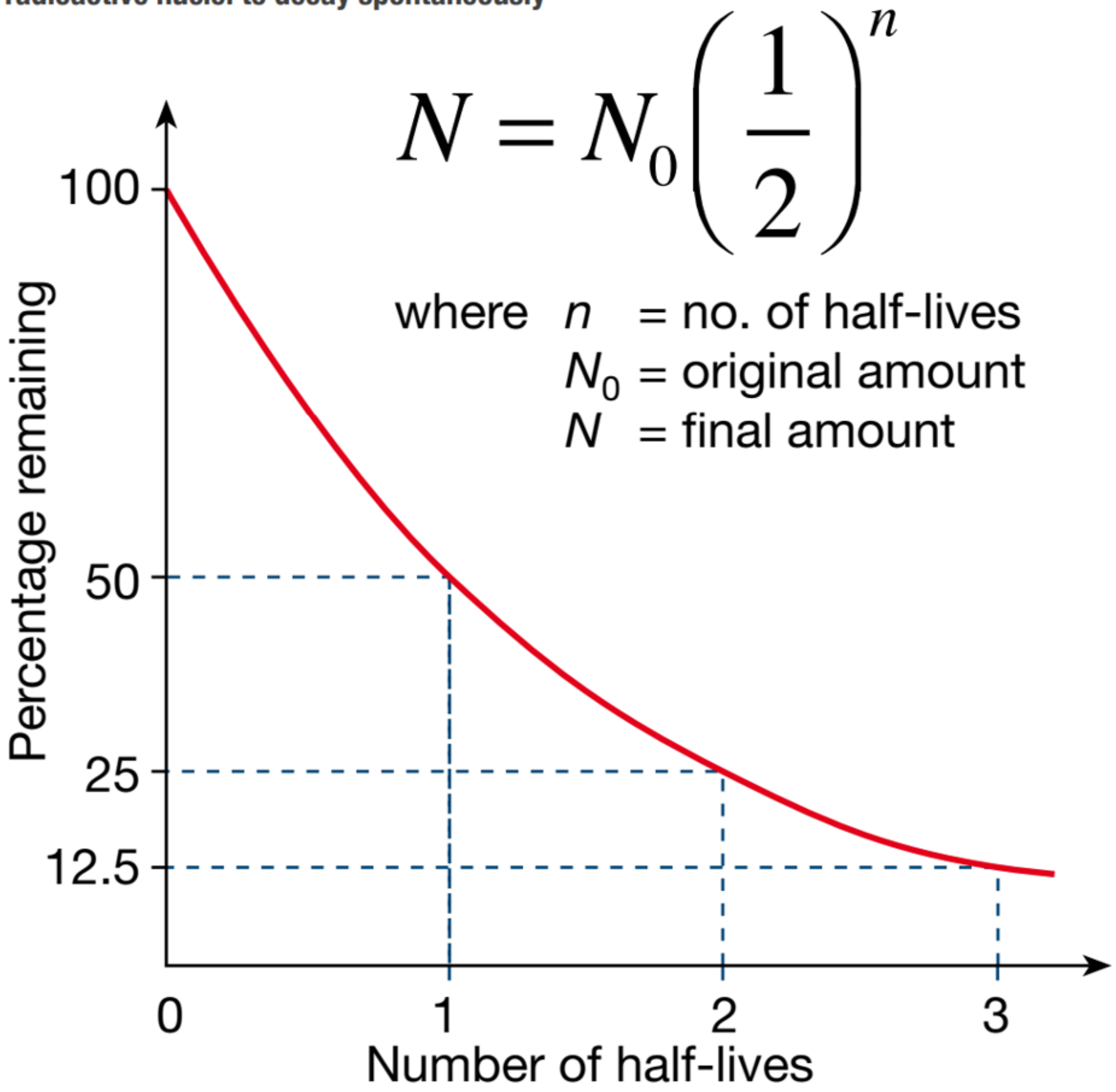
Cumulative Activity

- When a radionuclide is implanted it is often useful to know the amount of activity given off by that sample over its entire lifetime.
- Cumulative activity is equal to

$$(\hat{A} = A / \lambda = 1.443 * T_{1/2} * A)$$

Half-life

Half-life is the time taken for half the radioactive nuclei to decay spontaneously



Becquerels

Decay is measured in Becquerels (Bq)
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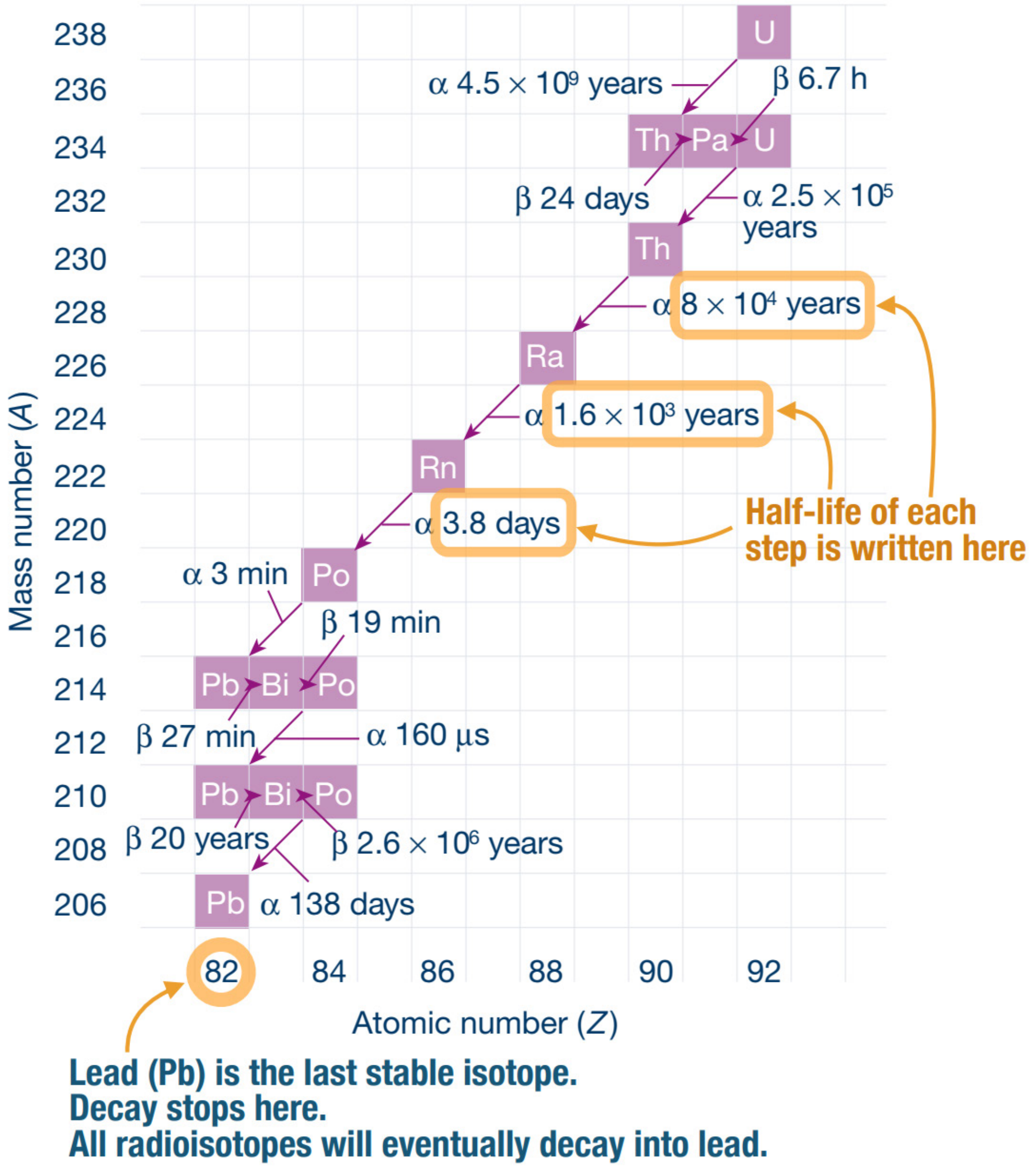
Radioactive Decay Chains

- If we consider successive radioactive samples (that is, a parent nuclei decays to a progeny nuclei which then decays to another product) then a state of equilibrium may be attained

The rate of change dN_D / dt in number of daughter nuclei D equals to the supply of new daughter nuclei through the decay of P given as $\lambda_P N_P(t)$ and the loss of daughter nuclei D from the decay of D to G given as $-\lambda_D N_D(t)$

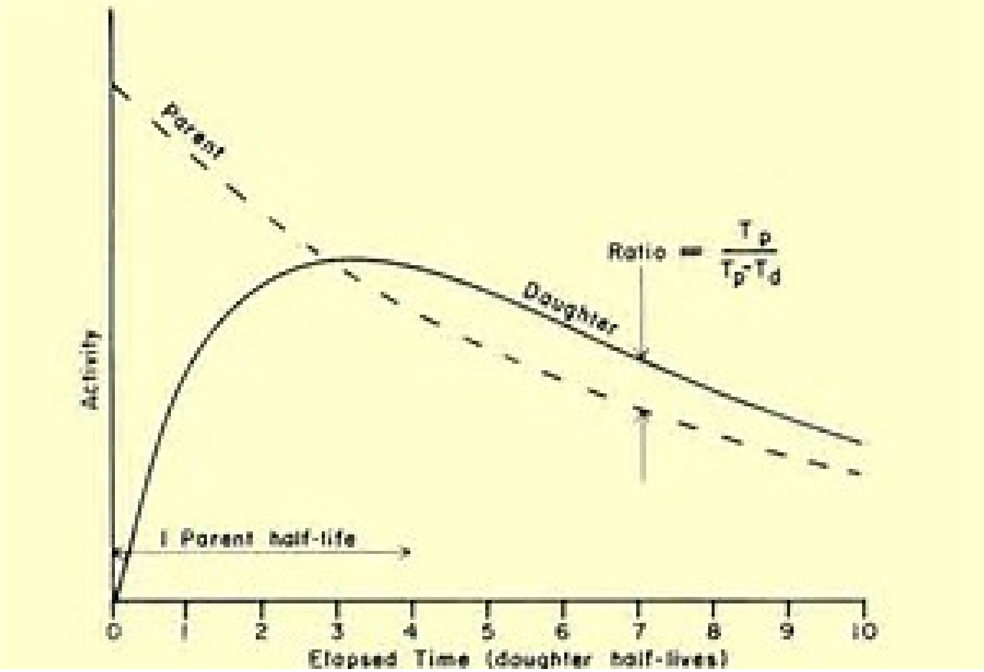
$$\frac{dN_D}{dt} = \lambda_P N_P(t) - \lambda_D N_D(t) = \lambda_P N_P(0)\, e^{-\lambda_P t} - \lambda_D N_D(t)$$

- There are two cases in which equilibrium may be achieved and both require that the half-life of the parent be larger than the half-life of the progeny



Transient Equilibrium

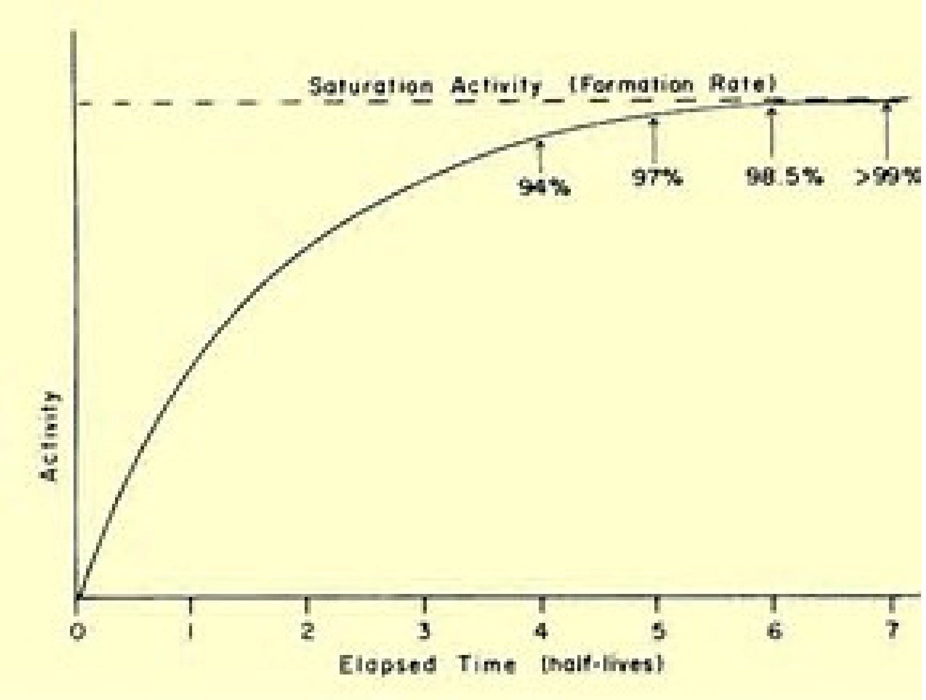
- This occurs when the parent half-life is only slightly larger than the progeny's (by a factor of 10 or so).
- In this case, the activity of the daughter rises dramatically, and then matches the activity decline of the parent as the population of radioactive atoms in the progeny becomes dependent on being fed by the parent population.
- At transient equilibrium, the daughter activity is related to the parent activity by:



$$A_2 = A_1\left(\frac{T_1}{T_1 - T2}\right)$$

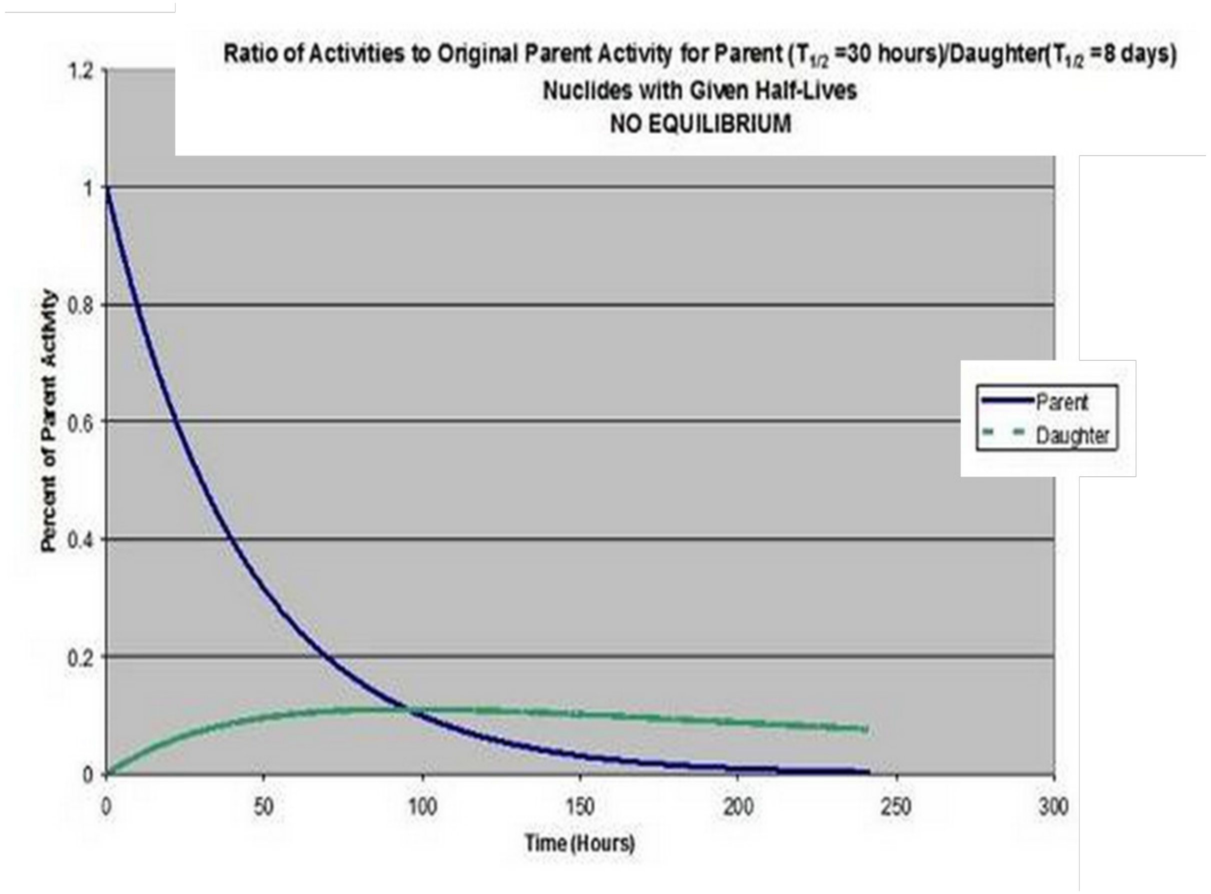
Secular Equilibrium

- At secular equilibrium, daughter activity is equal to parent activity (A_2=A_1).
- This occurs when the parent half-life is much greater than the progeny's (by a factor of 1000 or so).
- In this case, the daughter activity rises dramatically to meet the parent activity, but then attains a state of equal and dependent activity equilibrium.



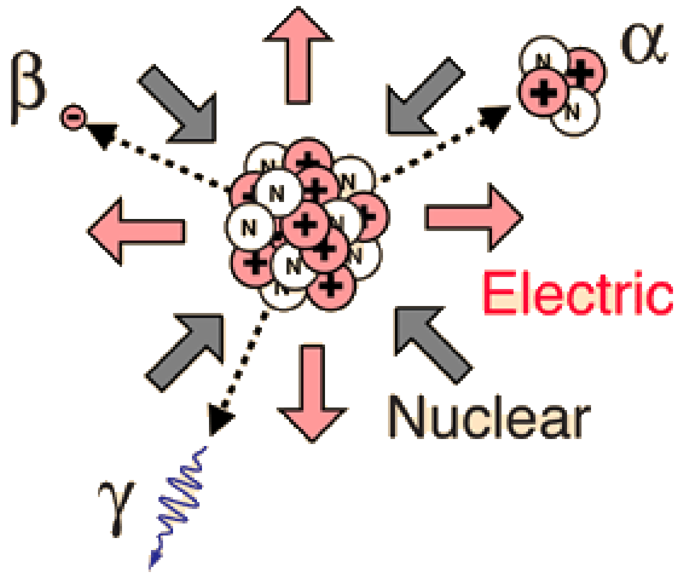
No Equilibrium

- This occurs if the daughter's half-life is greater than the parent's



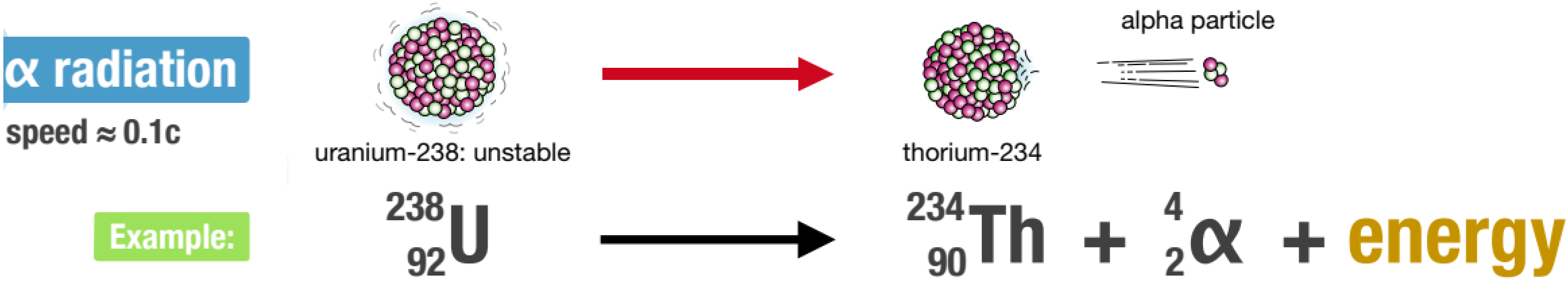
Radioactive Equilibrium Summary

❑ For $\lambda_D < \lambda_P$ or $(t_{1/2})_D > (t_{1/2})_P$ General relationship (no equilibrium)	$\frac{\mathcal{A}_D}{\mathcal{A}_P} = \frac{\lambda_D}{\lambda_D - \lambda_P}\left\{1 - e^{-(\lambda_D - \lambda_P)t}\right\}$
❑ For $\lambda_D > \lambda_P$ or $(t_{1/2})_D < (t_{1/2})_P$ Transient equilibrium for $t >> t_{max}$	$\frac{\mathcal{A}_D}{\mathcal{A}_P} = \frac{\lambda_D}{\lambda_D - \lambda_P}$
❑ For $\lambda_D >> \lambda_P$ or $(t_{1/2})_D << (t_{1/2})_P$ Secular equilibrium	$\frac{\mathcal{A}_D}{\mathcal{A}_P} \approx 1$



Types of Decay

Alpha Decay

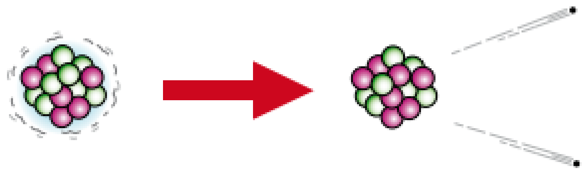


- Electrostatic repulsion between protons > Strong Nuclear Force
- The disintegration energy is given to the two nuclei as kinetic energy
- Atomic number decreases by 2 $Z_{daughter} = Z_{parent} - 2$
- Mass number decreases by 4 $A_{daughter} = A_{parent} - 4$

Beta- (Negatron) Decay

β radiation

speed ≈ 0.9c



$$Z_{daughter} = Z_{parent} + 1$$

$$A_{daughter} = A_{parent}$$

Example:

$${}^{14}_{6}C \rightarrow {}^{14}_{7}N + {}^{0}_{-1}\beta + \bar{\nu} + \text{energy}$$

neutron changes into a proton

low-energy electron ejected from the nucleus

antineutrino
accounts for the lower-than-expected energy of the β particle. Always released with β particles.

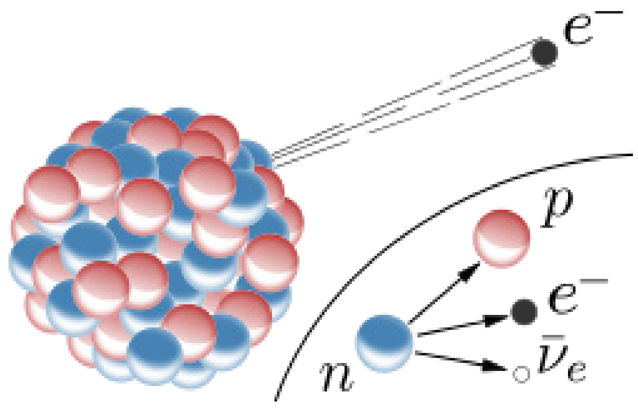
- A neutron releases an electron and leaves behind a proton

$${}^{1}_{0}n \rightarrow {}^{1}_{1}p + e^{-} + \nu$$

(neutron) (proton) (electron) (neutrino)

Beta- (Negatron) Decay

- This decay occurs typically when the radioactive sample has a high neutron to proton ratio therefore lying above the line of stability
- An example is Co-60 decay to Ni-60 in an excited state, which then immediately decays by gamma emission



$$ {}^{60}_{27}\mathrm{Co} \rightarrow {}^{60}_{28}\,\mathrm{Ni} + \beta^- + \tilde{\nu} + Q $$

Electron Capture

- A special form of beta decay
- When an atom contains too many protons in the nucleus for stability, one can combine with an inner-shell electron (K-capture)
- The result of this reaction is the conversion of a proton and electron into a neutron.
- i.e. orbital electrons is "captured" by the nucleus causing one of the protons to change state to a neutron
- This process competes with positron decay when there is not enough energy to fulfill positron emission's threshold of 1.022 MeV

$$^{A}_{Z}\mathrm{X} + ^{0}_{-1}\mathrm{e} \rightarrow ^{A}_{Z-1}\mathrm{Y} + \nu + Q$$

$$^{1}_{1}p \quad + \quad e^{-} \quad \rightarrow \quad ^{1}_{0}n \quad + \quad \nu$$

(proton) (electron) (neutron) (neutrino)

$$^{125}_{53}\mathrm{I} + ^{0}_{-1}\mathrm{e} \rightarrow ^{125}_{52}\mathrm{Te} + \nu + Q$$

Beta+ (Positron) Decay

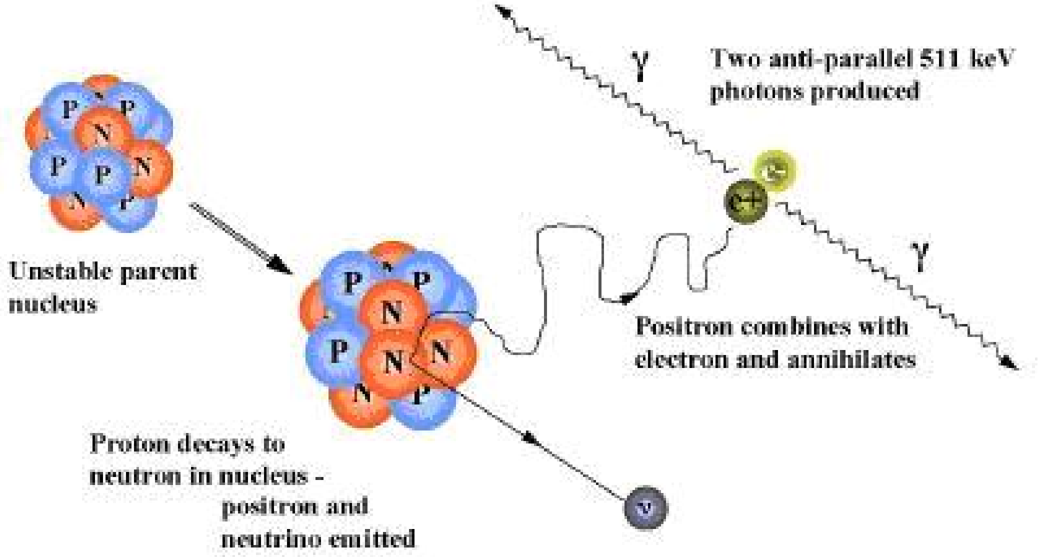
- There will be a continuous spectrum of energies for the positrons.
- Has a threshold of 1.022 MeV (rest mass of two electrons) for energy conservation
- A positron has the same mass as an electron but is opposite in charge. As it travels, it slows down through interactions with other atoms in the vicinity until it annihilates with any nearby electron
- The destruction of the electron and positron creates two gamma rays, each with energies of 0.511 MeV and emitted in opposing directions

$$^{A}_{Z}\mathrm{X} \rightarrow ^{A}_{Z-1}\mathrm{Y} + \beta^{+} + \nu + Q$$

$$^{1}_{1}p \rightarrow ^{1}_{0}n + e^{+} + \nu$$

(proton) (neutron) (positron) (neutrino)

$$^{18}_{9}\mathrm{F} \rightarrow ^{18}_{8}\mathrm{O} + \beta^{+} + \nu + Q$$

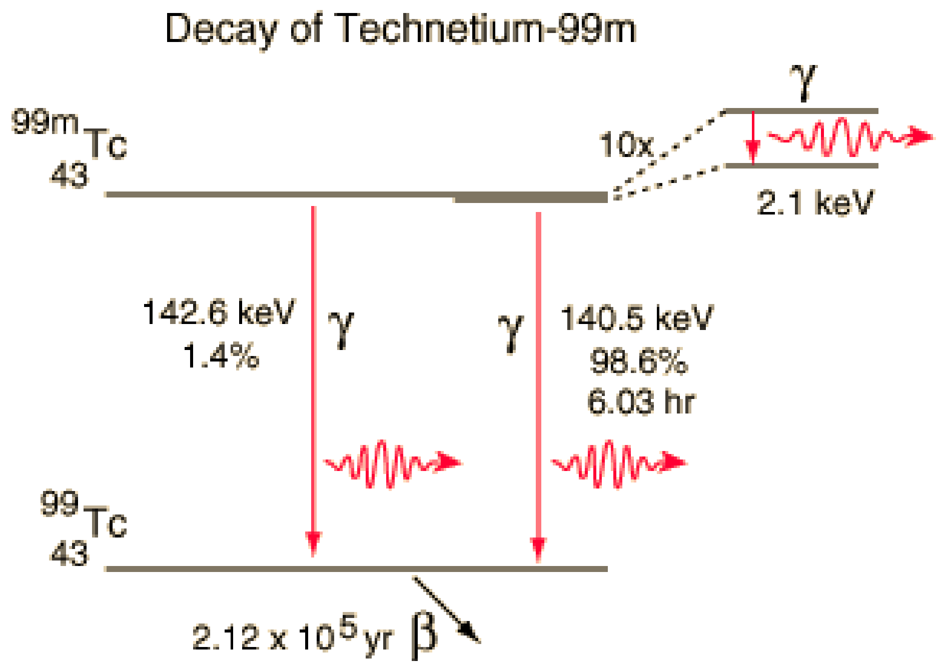


Gamma Decay

$$^{Am}_{Z}\mathrm{Y} \rightarrow ^{A}_{Z}\mathrm{Y} + \gamma$$

$$^{99m}Tc \rightarrow ^{99}Tc + \gamma\,(0.14\ MeV)$$

- Gamma decay is the release of high energy photons from an excited nucleus.
- After undergoing decay through one of the methods above, the nucleus may be in a higher energy state (similar to an excited electron existing in a higher energy shell). The nucleus releases this energy is a gamma ray.
- This is a **very common** occurrence, although the 'm' (standing for metastable state) is only used when the nucleus remains in the excited state for some period of time
  - Called an isomeric transition



Internal Conversion

- Nuclear deexcitation energy is absorbed by an orbital electron
- The electron is emitted, leaving a shell vacancy
- An higher level orbital electron fills the vacancy, resulting in the emission of characteristic X-rays or Auger electrons
    - Auger electrons occur when the energy typically released as a characteristic X-ray is transferred to another orbital electron, which is then ejected from the atom

Spontaneous Fission

- A high atomic mass nucleus spontaneously splits into two nearly equal fission fragments
- 2-4 neutrons are also emitted
- Competes with alpha decay for elements with A>230 and $Z^2/A > 235$