Math Review, Quantities & Units

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Calculator Use

- The main thing is to start using now the calculator that you will use in the clinic and the test
- Know how to use each of its functions
- If you don't know how to use the calculator that you have, either get a newer, easier one, or read the instructions and practice.

Calculator Use

- Whatever calculator you have right now should have the following functions:
  - log
  - ln
  - e^x (aka exp[x])
  - π
  - x^2
  - $\sqrt{x}$ (aka sqrt)
  - sin
  - cos
  - tan
  - Exp (aka EE)
  - 1/x

Calculator Use

2 * 3 + 4 = ??

2 * (3+4) = 14

A scientific calculator that correctly evaluates mathematical expressions will give an answer of 10.

Calculator Use

Calculate square root of 27

Press: 27 = (or 2^{nd}F x^2 27)=

Result: 5.196152423

Calculate cube root of 1234

Press: 234 2^{nd}F = (or 3 2^{nd}F ^ 1234) =

Result: 10.72601467

Calculator Use

Calculate the square of 15

Press: 15 x^2 =

Result: 225

Calculate 7 to the 4th power.

Press: 7 y^x 4 = (or 7 ^ 4) =

Result: 2401

Calculator Use

Find the 5th root of 243.

Press: 243 2ndF y^x 5 = (or 5 2ndF ^ 243)=

(any other alternatives?)

Result: 3

Calculate logarithm of 31.62

Press: 31.62 log = (or log 31.62)=

Result: 1.499961866

Calculator Use

Calculate the natural logarithm of 31.62

Press: 31.62 ln = (or ln 31.62)=

Result: 3.453789832

Calculate $10^{4.7}$

Press: 10 y^x 4.7 = (or 10 ^ 4.7)=

Result: 50118.72336

Calculator Use

Calculate $e^{3.42}$

Press: 3.42 2ndF ln = (2nd ln 3.42) =

Result: 30.56941502

Calculate $e^{-3.42}$

Press: -3.42 2ndF ln = (2ndF ln 3.42)=

Result: 0.032712434

Calculator Use

Calculate the reciprocal of 974.87

Press: 974.87 $2^{nd}F$ x^2= (or 974.87 x^{-1})=

Result: 0.001025777

Increase 38 by 15%.

Press: 38 + (0.15 x 38) = (or 38 x 1.15)=

Result: 43.7

Calculate 47% of 219.

Press: 219 X 0.47 =

Result: 102.93

General Mathematics

Exponentials & logarithms

$e^{-a} = 1/e^a$ $\ln(1/x) = -\ln x$

$e^b e^c = e^{(b+c)}$

Circumference of a circle, $C = 2\pi r$

Area of a circle, $A = \pi r^2$

Area of a sphere, $A = 4\pi r^2$

Percentages

What is 30% of 432?

129.6

Prescription written for 4500 cGy. You decide to treat to the 98% line. What is the dose at the 100% line?

4592 cGy

Your hot spot was at 102%. When you treat to the 98% line, what is the dose at the hot spot?

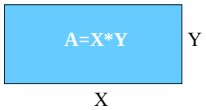
4683 cGy

Basic Math -Equations

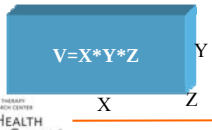
- Area

Volume

circle



sphere



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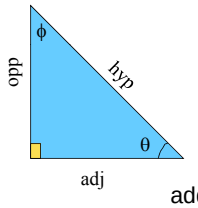
Pythagorean (right triangle)

- SOH CAH TOH

$opp^2 + adj^2 = hyp^2$



$\sin \theta = opp/hyp$
$\cos \theta = adj/hyp$
$\tan \theta = opp/adj$

The 3 angles must add add up to = 180º. So, $\theta + \phi = 90º$.

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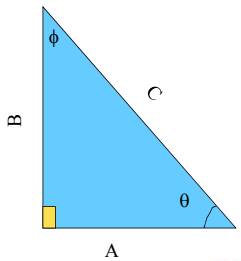
Pythagorean Example



- $A^2 + B^2 = C^2$
- If A = 3 & B = 4, what is C?

  C = 5
- What is ϕ?

  $\cos \phi = B / C$

  $\cos \phi = 4 / 5$

  $\cos^{-1}(\cos \phi) = \cos^{-1}(4 / 5)$

  $\phi = \cos^{-1}(4 / 5) = 36.9º$

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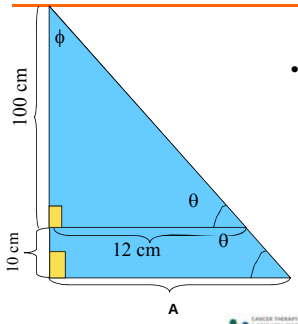
Similar Triangles

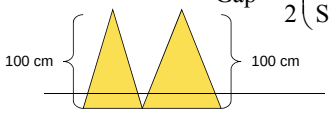


- A field width is 12 cm at 100 cm SSD. Determine the field width when projected at 10 cm beyond the surface.

Gap Calculation

- Calculate the gap needed on the skin for 2 fields where the first field has a length of 25 cm at an SAD of 100 cm and the second field has a length of 35 cm at an SAD of 100 cm. the depth of the match is 10 cm.

$$\text{Gap} = \frac{d}{2}\left(\frac{\text{field}_1}{\text{SSD}_1} + \frac{\text{field}_2}{\text{SSD}_2}\right)$$



Depth of matching (d) = 10 cm

HVL

- Determine the exposure rate of a source at 1 meter when the 10 cm of Pb has been put between the source and the detector. The detector reads 100 mR/h with no absorber in place. The HVL is 20 mm of Pb.

$$\mu = \frac{\ln 2}{\text{HVL}}$$

$$X = X_0 \exp(-\mu x) = 100 \frac{\text{mR}}{\text{h}} \exp\left(-\frac{\ln 2}{2\text{cm}} \times 10\text{cm}\right)$$

SI Base Units

In the SI system there are seven fundamental units and two supplemental units. Everything thing in the physical world can be described by these units.

The language of equations or objective science is spelled with only nine letters (seven base units and two supplemental units).

Seven Fundamental Units

Quantity	Unit name	Abbreviation
Length	meter	m
Mass	kilogram	kg
Time	second	s
Light intensity	candela	cd
Current	ampere	A
Temperature	Kelvin	K
Quantity of mass	mole	Mol

Supplemental Units

Quantity	**Unit name**	**Abbreviation**
plane angle	radian	rad
solid angle	steradian	sr

Prefixes

Prefix	Symbol	Factor
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
hecto	h	10^{2}
deka	da	10^{1}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}

Conversion

- How many cm are in 2 m?
- How many m are in 1 μm?
- How many m are in 1 Gm?

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Conversions

1 tesla (T) = 10^{4} gauss
1 joule (J) = 10^{7} erg
1 angstrom (Å) = 10^{-10} meter
1 mile (mi) = 1609 meter
1 curie (Ci) = 3.7 x 10^{10} becquerel
1 electron volt (eV) = 1.6 x 10^{-19} J
1 roentgen (R) = 2.58 x 10^{-4} C/kg
1 rad = 10^{-2} gray = 1 cGy
1 rem = 10^{-2} sievert = 1 cSv
1 calorie = 4.19 joule

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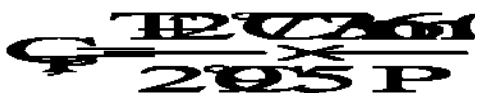
Unit Conversions

- Length: 1 meter = 3.3 feet
  2.54 cm = 1 inch
- Mass: 1 kg = 2.2 lb
  1 lb = 0.453592 kg (just reverse above)
- Time: 1 sec = 1/60 min = 1/3600 hr
- Temperature: °F = (1.8 * °C) +32
  Kelvin = °C + 273.15

Universal Constants

Avogadro's number, $N_A = 6.02 \times 10^{23}$ atoms/mol
Planck's constant, $h = 6.62 \times 10^{-34}$ Js
Velocity of light, $c = 3 \times 10^{8}$ m/s
Base of natural logarithms, e = 2.7183
Pi, $\pi = 3.14156$
Electronic charge, $q = 1.602 \times 10^{-19}$ C
Electron mass, $m_e = 9.11 \times 10^{-31}$ kg
Absolute Zero, 0 K = - 273.15 °C

Correction for Temperature & Pressure



$C_{T,P}$ Example

- The temperature is 20°C and the pressure is 733 mmHg. What correction factor do we need to use?

Conventional Units

Curie: rate of radioactive decay

Quantity of any radioactive nuclide which undergoes 3.7×10^{10} disintegrations per second.

$1Ci = 3.7 \times 10^{10}$ Bq or 37 MBq = 1 mCi

Roentgen: That quantity of x-rays or gamma radiation such that it produces in air charged particles of either sign equal to 2.58×10^{-4} C/kg.

Conventional Units

Rad: an ionizing radiation unit corresponding to an absorption of energy in any medium of 100ergs/g.

1 rad = 0.01 gray = 1cGy.

Gray: SI derived unit of absorbed dose of ionizing radiation

1Gy=1J/kg.

Conventional Units

Rem:

- Abbreviation for roentgen-equivalent man
- 1 rem=100 ergs/g absorbed energy=0.01 Sv
- Used only for radiation protection purposes

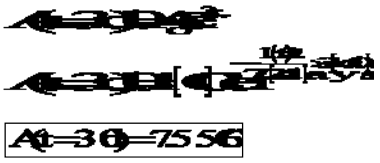
Sievert:

- SI unit or radiation dose equivalent
- One Sievert is the dose equivalent when the absorbed dose of ionizing radiation multiplied by the stipulated dimensionless factor is 1J/kg. 1 rem=0.01 Sv

Decay Example 1

The original activity of a Ir-192 source is 10 Ci. After 30 days, what is the activity?

– Note: the $t_{1/2}$ of Ir-192 is 74.2 days.



Exposure

- Want to know how much radiation is there
- Relates to indirectly ionizing radiation
- Measure of ionization produced in air by photons
- X = dQ/dm
- dQ is the absolute value of the total charge of ions of one sign (+ or -) produced in air when all the electrons (negatrons and positrons) liberated by photons in air of mass dm are completely stopped.
- Measured in Roentgens (or SI: C/kg)

Exposure

A small air filled cavity ion chamber collects a charge of 8 nC. The cavity volume is 0.200 cm^3 and the density of air is 0.0013 g/cm^3. What is the measured exposure? (hint: $m=\rho V$)

Exposure Rate

$\dot{X} = X/t$

$\dot{X} = A\,\Gamma / r^2$

A = activity (mCi)

Γ = exposure rate constant

(R cm^2/mCi-hr)

r = distance from source (cm^2)

Units

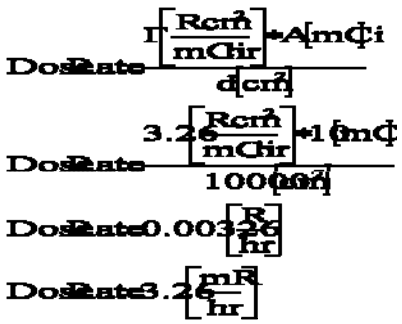
- By watching your units can actually help you solve problems that you aren't sure of the equations of.
- Example:

  What is the exposure rate at a distance of 1 meter for a 10 mCi source of Cs-137?

  Note: Cs-137 has a Γ of 3.26 R cm^2/ mCi hr.

Units

- Write out what all the problem has given us
  - DR = ?
  - d = 1 m = 100 cm
  - A = 10 mCi
  - Γ = 3.26 R cm^2/ mCi hr



Radiation Exposure

- Charge produced by ionizing electromagnetic (EM) radiation per unit mass of air.
  - C/kg in SI units (International System of Units).
  - 1 Roentgen (R) = 2.58 x 10^{-4} C/kg of air.
- Valid for photon energies up to 3 MeV.
- **Fluence** = number of photons passing through a unit cross-sectional area.
- **Fluence rate (flux)** = number of photons passing through a unit cross-sectional area per unit time.

Absorbed Dose

Amount of radiation that is absorbed

Energy imparted to matter (by electromagnetic radiation or particulate radiation) per unit mass

D=dE/dm

Units: rad, cGy;

1 Gy = 1 J/kg

100 rads = 1 Gy

1cGy = 1 rad

Absorbed Dose

Absorbed Dose depends on:

1. Energy of the radiation
   (as the energy of the radiation decreases, then absorbed dose increases)
2. Material of mass
   (as z of material increases, so does absorbed dose)

Absorbed Dose

- The radiant energy of the photons entering a 100 gram mass is 0.1 joules. The radiant energy of the photons leaving the mass is 0.05 joules. What is the absorbed dose within the mass?

F-factor

AKA the rad to Roentgen conversion factor

$D_{air} = F^* X$

F = 0.876 rads/R

Relates exposure in air to dose in air

The f-factor depends on:

- The energy of incident photon for all materials except air
- Depends on atomic number of absorber

F-factor

- What is the dose in air for an exposure of 1000 mR?