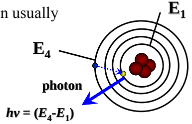
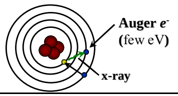
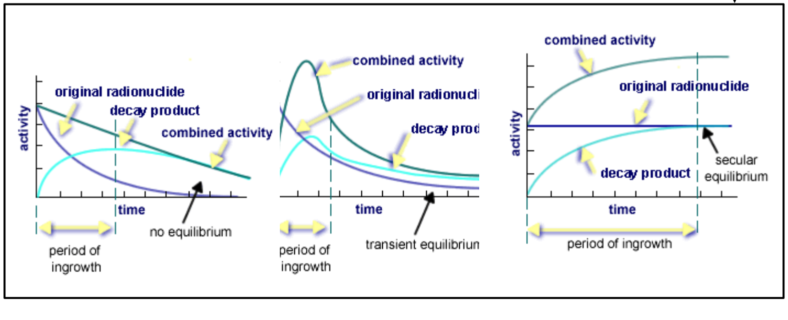
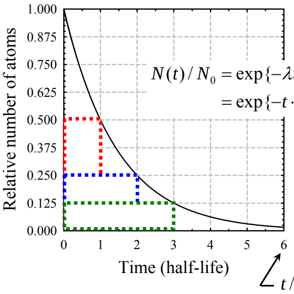
HSCI 312 – Exam 2
Equation Sheet

Term	*Definitions*
Mass Defect	Δ*M* equals the sum of the masses of reactants minus sum of the masses of the products
Conservation of Energy	Total energy (*E*) and momentum (*p* = *mv*) is conserved in all nuclear reactions. Kinetic energy may or may not be conserved. Mass may or may not be conserved.
Binding Energy of the Nucleus	When two elementary particle or atoms combine to form a new atom, the reaction energy is termed *binding energy of the nucleus*. The energy required to hold the nucleus together is the same as the energy required to tear the nucleus apart. Nuclear binding energy is much larger than electron binding energy (i.e. can be neglected in Q value calculations).
Reactions (Chemical vs. Nuclear)	Chemical reactions occur when the electron cloud associated with one atom reacts with the electron cloud of another atom (1 to 10 eV). Nuclear reactions occur when nucleons (neutrons and/or protons) interact: 1) Nucleus of two atoms interact; 2) Elementary particle interactions with nucleus; 3)Elementary particle interacts to form new atom (MeV)
Q-value	When two or more particles, ions or atoms interact to form two or more other particles, ions or atoms, the *reaction energy* is termed the *Q-value.* Smaller Q values mean harder to remove the neutron and thus the nucleus is more stable. - How to handle *ions*… Q value computed before ion loses or acquires electron and becomes electrically neutral (explained below). - In many nuclear reactions, atoms are “born” as positive or negative ions (too many or too few orbital electrons): ${}^{38}_{17}\mathrm{Cl} \to {}^{38}_{18}\mathrm{Ar}^{+} + e^{-} + \bar{\nu}$. In this case, convert the ion to a neutral atom by: 1) adding an electron to both sides of equation; then 2) Combine the ion with an existing electron on the same side of the equation; then 3) subtracting an electron from both sides of the equation… resulting in ${}^{38}_{17}\mathrm{Cl} \to {}^{38}_{18}\mathrm{Ar} + \bar{\nu}$
Fission vs. Fusion	Fission relies on neutrons to split heavy nuclei and yields ~ 200 MeV per reaction; fusion joins lighter nuclei and yields ~ 30 MeV per reaction…fusion produces less energy but eliminates uncontrolled chain reactions and radioactive waste, and is thus desirable
Excited State	Nuclei in an excited state spontaneously (and rapidly) transition back to the ground state. The energy appears as a high energy photon (γ-ray).
Gamma Ray	Gamma-rays are emitted from the nucleus when a nucleon transitions from an excited state to the ground state..Typical energies are from 0.01 MeV to 10 MeV
X-Ray	X-rays are formed when: Electrons are accelerated in any magnetic field; Photons generated when an electron is accelerated in the electromagnetic field associated with an atom are termed *bremsstrahlung* (or *bremsstrahlung x-rays*)
Characteristic X-Ray	Characteristic x-rays are emitted when electrons transition from an excited state to a lower energy state. In EC decay, the “captured” electron usually comes from the innermost shell (i.e., an energy state “closest” to the nucleus). Electrons in outer shells (higher energy states) fill the vacancy, usually emitting a photon. Photon energy equals the energy different between the two states, and are called *characteristic x-rays.* 
Auger Electron	When an electron in a higher orbit fills a vacancy in a lower orbit, transition energy may be transferred to another orbital electron (e.g., an intra-atom photoelectric effect). Electrons emitted through this process are referred to as *Auger electrons.* 
Neutron Interactions/ Scattering	When a neutron interacts, the nucleus is either left in the ground state where kinetic energy is conserved (*elastic scattering*) or an excited state where kinetic energy is not conserved (*inelastic scattering*). *Capture Scattering*: neutron is absorbed into the nucleus. Nucleus emits a neutron (not necessarily the same one) to return to the ground state. May be either an elastic or inelastic reaction. *Potential Scattering*: incident neutron scatters off entire nucleus – analogous to diffraction of neutron wave by nuclear potential. Always elastic.
Thermal Neutrons	Neutrons can be slowed through elastic collisions with the nuclei of atoms; The fraction of the energy (*f*) lost by a neutron in a head-on elastic collision is given by the formula (where *mn* is the mass of the neutron and *M* is the mass of the target atom) $f = \frac{4m_n M}{(m_n + M)^2}$
Radioactive Decay	Radioactive decay occurs when the nucleus of an atom *spontaneously* changes and one or more particles are emitted: 1) **Q value must be greater than zero for reaction to occur spontaneously! 2)** Fission and fusion are *not* examples of radioactive decay because these reactions do not occur spontaneously. However, they are examples of *nuclear reactions.*
Neutrino/ Anti-Neutrino	To conserve angular momentum of nucleus during β-decay, *antineutrino* must carry off some kinetic energy. The neutrino/anti-neutrino has 0 charge, 0 mass (rest mass energy a few eV), and Intrinsic spin ½. The energy released in β-decay is shared between electron and antineutrino. Thus, electrons appear with a spectrum of kinetic energies.
Stochastic	*Truly* stochastic phenomena: 1) Exact location or momentum of an orbital electron; 2) Interactions of radiation with matter; 3) Radioactive decay. *Seemingly* stochastic phenomena: 1) Coin flip; 2) Brownian motion of atoms; 3) Weather… These phenomena *appear* stochastic because they are chaotic. applies to ***SINGLE EVENT***
Deterministic (non-stochastic)	Describes state where we can reliably and accurately predict the outcome of a process: 1) Motion of planets around the sun; 2) Time it takes for an apple to fall from a tree; 3) Death and taxes. The fact that you will die someday is deterministic. However, the exact time when you’ll die is a seemingly stochastic event. ***applies to MEAN and VARIANCE***
Secular vs. Transient Equilibrium	*Transient*: $\lambda_2 - \lambda_1 > 0$ (daughter decays *slightly* faster than parent). *Secular*: $\lambda_2 - \lambda_1 >> 0$ (daughter decays *much* faster than parent and the activity of both isotopes is the same, $\lambda_1 N_1 = \lambda_2 N_2$). *No equilibrium*: $\lambda_1 << \lambda_2$
Bateman Equation	System of differential equations used to solve for activities in decay chains when initially only parent is present.
Cosmogenic Radionuclides	Interaction of energetic cosmic rays (mostly protons) interact in the atmosphere, sea or earth. Capture of secondary neutrons creates new radioactive isotopes. Most prominent of the cosmogenic radionuclides are tritium (^{3}H) and (^{14}C).
Primordial Decay Series	Each of the naturally occurring radioactive nuclides with Z > 83 is a member of one of three long decay chains. 1) Thorium (Th), 2) Uranium (U), and 3) Actinium (Ac) series; Decay by α or β⁻ emission; Decay constant of the first member of the decay chain is very short, which means a very long half-life (billions of years); Daughter products are in *secular equilibrium* with parent (each member of the decay chain has the same activity)
Singly Occurring Primordials	17 very long-lived radionuclides present in the environment, but not part of a decay chain. K-40 and Rb-87 are the two most significant in terms of human exposure.



Exponential decay curve (linear scale)

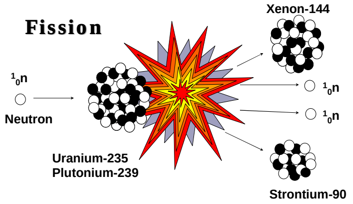


$$N(t)/N_0 = \exp\{-\lambda t\}$$
$$= \exp\{-t \cdot \ln 2 / T_{1/2}\}$$

$$\lambda t = 0.693 \cdot 6$$
$$= 4.158$$

$$t / T_{1/2} = 6$$

Fission



What makes a good neutron shield?

- Pick a material that maximizes the energy loss per collision $f = \frac{4m_1 m_2}{(m_1 + m_2)^2}$

Hydrogen (1 amu): $f = \frac{4 \cdot 1}{(1+1)^2} = 1\ (100\%)$

Carbon (12 amu): $f = \frac{4 \cdot 12}{(1+12)^2} = 0.284\ (28.4\%)$

Lead (206 amu): $f = \frac{4 \cdot 206}{(1+206)^2} = 0.0192\ (1.92\%)$

Light elements (**e.g., water**) make better neutron shields than heavy elements!

Specific activity (selected radioisotopes)

Isotope	Half-life	unit	Specific Activity TBq/g	Specific Activity Ci/g
^{3}H	12.35	y	357	9,649.71
^{14}C	5730	y	0.165	4.46
^{18}F	109.74	m	3.52E+06	95,145,600.00
^{22}Na	2.602	y	231	6,243.93
^{32}P	14.29	d	1.06E+04	286,518.00
^{60}Co	5.271	y	41.8	1,129.85
^{90}Sr	29.12	y	5.05E+00	136.50
^{90}Y	64	h	2.01E+04	543,303.00
^{131}I	8.04	d	4.59E+03	124,067.70
^{137}Cs	30	y	3.22E+00	87.04
^{192}Ir	74.02	d	3.40E+02	9,190.20
^{226}Ra	1600	y	3.66E-02	0.99

$$Decay\ Constant = \lambda = Probability\ per\ unit\ time\ an\ atom\ decays$$

$$\triangleright\ \lambda = \sum_i \lambda_i = \lambda_1 + \lambda_2 + \lambda_3 \{competing\ processes\}$$

$$\#\ of\ Atoms = N(t) = N_0 e^{-\lambda t} ....expected\ or\ avg.\ \#\ of\ atoms\ at\ time,\ t$$

$$Half - Life = T_{1/2} = -\frac{ln(0.5)}{\lambda} = \frac{ln2}{\lambda} \cong \frac{0.693}{\lambda}$$

$$Avg.\ Lifetime = T_{av} \equiv \int_0^\infty dt\ tp(t) = \int_0^\infty dt\ t \cdot \lambda e^{-\lambda t} dt = \frac{1}{\lambda}$$

$$Rate\ of\ Decay = \frac{dN(t)}{dt} = -\lambda N(t)$$

$$Activity = A \equiv \lambda N$$

$$\triangleright\ A(t) = A_0 e^{-\lambda t}$$

$$\triangleright\ 3.7 \times 10^{10}\ Bq = 1\ Ci\ (1\ g\ of\ {}^{226}Ra)$$

$$\triangleright\ 1\ Bq = 1\ dps = 1\frac{decay}{second} = 1\frac{disintegration}{second}$$

$$Specific\ Activity = \hat{A} \equiv \frac{A}{m} ....unique\ for\ each\ isotope$$

$$Part.\ emission\ rate = \frac{particles}{decay} \times \frac{decays}{second} = \frac{particles}{second}$$

$$Energy\ release\ rate = \sum_i \left(\frac{particles}{second} \times \frac{Energy}{particle}\right)_i = \frac{Energy}{second}$$

$$Decay\ w/\ Production = N(t) = N_0 e^{-\lambda t} + \frac{Q_0}{\lambda}\left[1 - e^{-\lambda t}\right] ....production\ fluctuating$$

$$= Q_0 - \lambda N(t) ....production\ constant$$

$$= N_{eq} = \frac{Q_0}{\lambda} ....equilibrium$$

$$Saturation\ Activity = As\ t \to \infty,\ A_{eq} = Q_0 .....unit : atoms / second$$

$$\triangleright\ ...\ from\ A(t) = A_0 e^{-\lambda t} + Q_0\left[1 - e^{-\lambda t}\right]$$

$$\triangleright\ Q_0 = production\ rate$$

$$\triangleright\ A(t) = A_{eq}\left(\frac{1}{2}\right)^{t/T_{1/2}} ...after\ saturation\ (removed\ from\ reactor)$$

$$Radiodating = \frac{N(t)}{N(0)} = e^{-\lambda t} \to t = -\frac{1}{\lambda} ln\left[\frac{N(t)}{N(0)}\right] = \frac{1}{\lambda} ln\left[\frac{N(0)}{N(t)}\right]$$

$$\triangleright\ {}^{14}C\ radiodating\ very\ common....good\ for\ \sim 57{,}000\ yrs$$

$$\left.\begin{aligned} &\triangleright\ t = \frac{1}{\lambda} ln\left[\frac{N(0)}{N(t)}\right] \\ &\triangleright\ t = -\frac{1}{\lambda} ln\left[\frac{mass(t)}{mass(0)}\right] \\ &\triangleright\ t = \frac{1}{\lambda} ln\left[\frac{activity(0)}{activity(t)}\right] \end{aligned}\right\} Any\ quantity\ proportional\ to\ N\ is\ valid$$

$$\triangleright\ t = \frac{1}{\lambda} ln\left[\frac{N(0)/N_s}{N(t)/N_s}\right] ....relies\ on\ stable\ isotope\ to\ estimate\ N(0)$$

β^- *decay* $= {}^{A}_{Z}P \rightarrow {}^{\;\;A}_{Z+1}D^+ + \beta^- + \bar{\nu}$ *or* ${}^{A}_{Z}P + e^- \rightarrow {}^{\;\;A}_{Z+1}D + \beta^- + \bar{\nu}$

- ▷ *Neutron is converted into a proton; electron* (β^-) *and anti-neutrino are emitted*
- ▷ *Decay energy shared between* β^- *and anti-neutrino;* β^- *particles appear with distribution of kinetic energies*
- ▷ *Anti-neutrino has little if any mass (assume zero mass for Q-value calculation)*
- ▷ *Example:* ${}^{32}_{15}P \rightarrow {}^{32}_{16}S^+ + e^- + \bar{\nu}$ *or* ${}^{32}_{15}P \rightarrow {}^{32}_{16}S + \bar{\nu} \Rightarrow Q = (31.97390716 - 31.97207069)\cdot\frac{931.494\ MeV}{1u} = 1.712\ MeV$
- ▷ $E_{max} = 1.712\ MeV + 0.511\ MeV = 2.222\ MeV$

β^+ *decay* $= {}^{A}_{Z}P \rightarrow {}^{\;\;A}_{Z-1}D^- + \beta^+ + \nu \rightarrow {}^{\;\;A}_{Z-1}D + e^- + \beta^+ + \nu$

- ▷ *Proton is converted into a neutron; positron* (β^+) *and neutrino are emitted*
- ▷ *Decay energy shared between* β^+ *and neutrino;* β^+ *particles appear with distribution of kinetic energies*
- ▷ *Neutrino, like the anti-neutrino, has little if any mass (assume zero mass for Q-value calculation)*
- ▷ β^+ *possible when* $\left(mass_{parent\ nuclide} \geq (2\cdot m_e) + mass_{decay\ product}\right)$
- ▷ *Example:* ${}^{22}_{11}Na \rightarrow {}^{22}_{10}Ne + e^+ + \nu \Rightarrow Q = \left\{M\left({}^{22}_{11}Na\right)\right\} - \left\{M\left({}^{22}_{10}Ne\right) + 2\cdot m_e\right\}\cdot\frac{931.494\ MeV}{1u} = 1.82\ MeV$
- ▷ β^+ will slow down and combine with an e^- to produce two 0.511 MeV annihilation photons

EC = *Electron Capture* $= {}^{A}_{Z}P \rightarrow {}^{\;\;A}_{Z-1}D^* + \nu \rightarrow {}^{\;\;A}_{Z-1}D + \nu + \gamma$

- ▷ *Orbital electrons have a small but non-zero probability they will spend some time inside the nucleus.*
- ▷ *A proton may, on occasion, capture one of these orbital electrons and be converted into a neutron*
- ▷ *EC and* β^+ *decay produce the same decay (daughter) product*
- ▷ *EC can occur as long as* $Q > 0$
- ▷ EC will produce conversion electrons, Auger electrons, and characteristic x-rays

γ *decay* $= {}^{A}_{Z}P^* \rightarrow {}^{A}_{Z}P + \gamma$ {*nucleus left in excited (*) state...excitation E released as photon*}

- ▷ $E_\gamma = \dfrac{Q}{1 + \dfrac{E_\gamma}{2M\left({}^{A}_{Z}P\right)c^2}}$
- ▷ $E_\gamma \leq\sim$ 10 *to* 20 *MeV and* $M\left({}^{A}_{Z}P\right)c^2 \sim 4000\ MeV$
- ▷ *Mass number (A) and atomic number (Z) stay the same*
- ▷ *Nucleus in excited state more massive than nucleus in ground state* ∴ *Exothermic*
- ▷ *Transition occurs very quickly (< 10-9 s)*
- ▷ *Example:* ${}^{97m}_{43}Tc \rightarrow {}^{97}_{43}Tc + \gamma \Rightarrow E_\gamma = Q\dfrac{1}{1 + \dfrac{20\ MeV}{2\cdot 4000\ MeV}} = \boxed{0.9975\cdot Q \cong Q}$

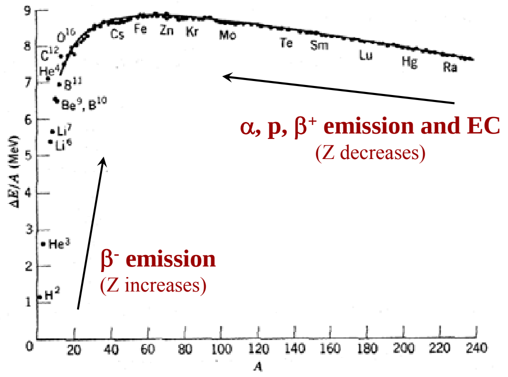
IC = *Internal Conversion* $= {}^{A}_{Z}P^* \rightarrow {}^{A}_{Z}P^+ + e^-$ {*excitation E released as* "conversion" *electron, ce*}

- ▷ *Process competes with* γ *decay; excited nucleus transfers excitation energy to an orbital electron*
- ▷ *Nucleus left in ground state, and atom carries a positive charge*
- ▷ *Electron kinetic energy* $= Q - BE_{electron}$
- ▷ *Example:* ${}^{97m}_{43}Tc \rightarrow {}^{97}_{43}Tc^+ + e^-$

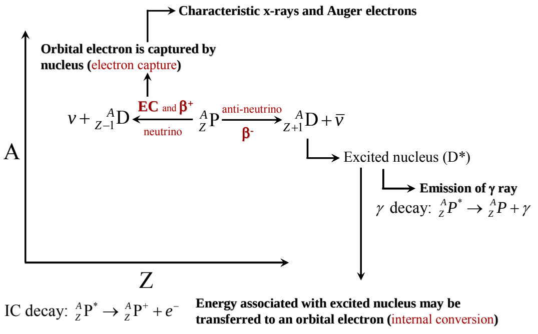
α *decay* $= {}^{A}_{Z}P \rightarrow {}^{A-4}_{Z-2}D^{2-} + \alpha^{2+} \rightarrow {}^{A-4}_{Z-2}D + 2e^- + \alpha^{2+} \rightarrow {}^{A-4}_{Z-2}D + \alpha$

- ▷ $E_\alpha = Q_\alpha \cdot \dfrac{M\left({}^{A-4}_{Z-2}D\right)}{M\left({}^{A-4}_{Z-2}D\right) + m_\alpha} = \left[M\left({}^{A}_{Z}P\right) - M\left({}^{A-4}_{Z-2}D\right) - m_\alpha\right]\cdot\dfrac{M\left({}^{A-4}_{Z-2}D\right)}{M\left({}^{A-4}_{Z-2}D\right) + m_\alpha}$
- ▷ *Nucleons tend to group together into subunits of* ${}^{4}He$ *nuclei*
- ▷ *Emission of an alpha particle* $({}^{4}He^{2+})$ *is a common mode of decay for heavier radioisotopes*
- ▷ *Alpha particle quickly loses energy and captures two electrons to become a neutrally charged He atom*
- ▷ *Example:* ${}^{226}_{88}Rn \rightarrow {}^{222}_{86}Ra + \alpha \Rightarrow E_\alpha = 4.870\ MeV\cdot\dfrac{222}{222+4} = 4.784\ MeV$

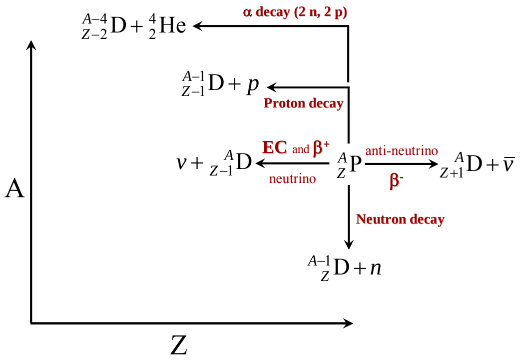
Radioactivity enhances nuclear stability



Positron (β^+) and negatron (β^-) decay



Neutron, proton and alpha decay



Mass Defect = {*mass of reactants*} − {*mass of products*}

- ▷ $\Delta M = \{M_A + M_B + ...\} - \{M_C + M_D + ...\}$
- ▷ *Example:* $H = \{1.00727u + 5.48579\times10^{-4}u\} - \{1.00782u\} = 1.4711\times10^{-8}u$

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Table 1.5 (*and Table A*.1) *Appendix B, Table B*.1

Nuclear BE ▷ *For a nucleus with Z protons and (A-Z) neutrons*

- ▷ $BE({}^{A}_{Z}X) = \left[Z\cdot M({}^{1}_{1}H) + (A-Z)\cdot m_n - M({}^{X}_{Z}X)\right]c^2$

$= \left[Z\cdot 1.0078250321u + (A-Z)\cdot 1.0086649233u - M({}^{X}_{Z}X)\right]\cdot\dfrac{931.494\ MeV}{1\ u}$

: ↗

Appendix B

- ▷ *Nuclear BE values often expressed on a per nucleon basis* $\Rightarrow \dfrac{BE({}^{A}_{Z}X)}{A}$

$$E = mc^2 = T + m_0c^2$$

Total energy ⤴ Kinetic energy ⤴ ↳ Rest mass energy

$\Delta E > 0$ (exothermic) $\Delta E < 0$ (endothermic)

↓ ↓

energy released **energy absorbed**

Total energy of system is conserved: $\Delta E - \Delta Mc^2 = 0$

$$\Delta E \rightarrow Q = \Delta Mc^2$$

$Q > 0$ (exothermic) $Q < 0$ (endothermic)

mass decreases **mass increases**

Atomic Nomenclature

- ▷ *Ground state:* ${}^{A}_{Z}X_N \rightarrow {}^{A}X$
- ▷ *Excited state:* ${}^{A}_{Z}X^* \rightarrow {}^{A}X^*$
- ▷ X = *Element (e.g., C or He)*
- ▷ Z = Atomic #
- ▷ N = *Neutron* #
- ▷ $A = (N + Z)$ = *Mass* #

Below:

...superscript denotes mass (u)

...subscript denotes charge

- ▷ $\left.\begin{cases} Proton: {}^{1}_{1}p \text{ or } p \\ Neutron: {}^{1}_{0}n \text{ or } n \end{cases}\right\} Nucleons$
- ▷ $\begin{cases} Electron: {}^{\;\;0}_{-1}e \text{ or } e^- \text{ or } \beta^- \\ Positron: {}^{\;\;0}_{+1}e \text{ or } e^+ \text{ or } \beta^+ \end{cases}$
- ▷ $\{Alpha: {}^{4}_{2}He \text{ or } {}^{4}_{2}He^{2+} \text{ or } \alpha$