Radiation Science Equation Sheet (Exam1)

Lecture #2: Units and Physical Quantities

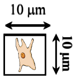
- A **physical quantity** characterizes event or process in terms suitable for numerical specification and manipulation. Length, time, volume, absorbed dose are all physical quantities.
- A **unit** is a selected reference sample of a physical quantity. Every physical quantity should be expressed as the product of a numerical value and a unit.
- Example **unit conversion**: Suppose we are given a dose rate in mGy/day and want to convert this to Gy/h.

$$2.3\frac{\text{mGy}}{\text{day}}\times\frac{\text{Gy}}{1{,}000\text{ mGy}}\times\frac{1\text{ day}}{24\text{ h}}=9.583\times10^{-5}\frac{\text{Gy}}{\text{h}}$$

- Common **Non SI Units** (see http://physics.nist.gov/cuu/Units/)
  - ångström (Å): 1 Å = 0.1 nm = 10^{-10} m
  - unified atomic mass unit (u): 1 u = 1.66054 × 10^{-27} kg
  - electron volt (eV): 1 eV = 1.60218 × 10^{-19} J
  - barn (b): 10^{-28} m2
  - curie (Ci): 1 Ci = 3.7 × 10^{10} Bq
  - roentgen (R): 1 R = 2.58 × 10^{-4} C/kg
  - rad (rad): 1 rad = 1 cGy = 10^{-2} Gy
  - rem (rem): 1 rem = 1 cSv = 10^{-2} Sv
- **Superposition principle** says that doses and dose rates that arise from different radiation sources can be added together. total dose rate (dr) $=\sum_i\{\text{dr}\}_i=\{\text{dr}\}_1+\{\text{dr}\}_2+\cdots$
- **Cell mass** (m), **volume** (V), **density** (ρ) equations

	SI prefixes	
Factor	**Name**	**Symbol**
10^{24}	yotta	Y
10^{21}	zetta	Z
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10^{1}	deka	da
0		
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	µ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a
10^{-21}	zepto	z
10^{-24}	yocto	y

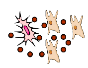
Cell Mass - Soft tissue in human body has ~ same density as water (1 g cm^{-3}). Since the human body is composed of cells, let's assume cells also have about the same density as water.



$$m=V\rho=(10^{-9}\text{ cm}^3/\text{cell})\times(1\text{ g/cm}^3)=10^{-9}\text{g/cell}$$

$$=10^{-9}\text{g/cell}\times\left(\frac{10^9\text{ng}}{\text{g}}\right)=1\text{ ng/cell}$$

How many cells in the human body?



$$\text{No. cells}=\left(\frac{\text{mass of human}}{\text{mass of cell}}\right)=\left(\frac{175\text{ lb}}{2.2\times10^{-12}\text{lb}}\right)=7.95\times10^{13}\text{cells}$$

Expect about 10^{11} to 10^{13} cells in human body

How long for radiation to pass through a cell?

An electron traveling at the speed of light passes through 10 µm of water (tissue):

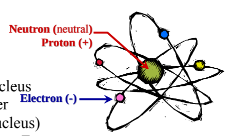
Distance (m) = time (s) × speed (m/s)

time = distance/speed

$$\text{time}=\frac{10\ \mu\text{m}\times\left(\frac{\text{m}}{10^6\ \mu\text{m}}\right)}{3\times10^8\ \frac{\text{m}}{\text{s}}}=3.3333\times10^{-14}\text{s}$$

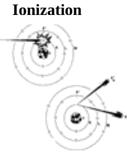
$$=3.3333\times10^{-14}\text{s}\times\left(\frac{10^{12}\text{ps}}{\text{s}}\right)=0.033333\text{ ps (pico seconds)}$$

Lectures #3 and #4: The Atom



- **1st known fission reactor** "built" about 1.7 billion years ago in Africa.
- Each **electron and proton** has a **charge** equal to 1.60217653 × 10-19 C.
- **Atom** uniquely identified by # of neutrons (N) and # of protons (Z) in the nucleus
  Z = atomic number…N = neutron number…A = (N + Z) = mass number
- **N/Z Ratios**: As Z increases, N/Z increases (more neutrons needed to bind nucleus)
- **Ionization:** process whereby one or more electrons are liberated from an atom; Z and A remain the same; atom becomes positively charged (because *e-* is removed)
- **Ionizing Radiation**: (1) Atomic or sub-atomic particles with sufficient kinetic energy to ionize matter. Electrons, positrons, neutrons, protons, alpha ($^4He^{2+}$) particles; (2) Very energetic photons (electromagnetic radiation); x-rays and γ–rays
- **Elements:** All atoms of *same element* have *same Z, but may have different A*.
- **Isotope (or nuclide):** Atom with specific # of protons and neutrons is termed a *nuclide*. Can either be ***stable*** (Z and *N* do not change) or ***unstable*** (Z or *N* spontaneously change). ***Unstable nuclides are radioactive.***
- **Isobar:** same mass # ($A = N + Z$) but different # of neutrons and protons; Examples: 14B, 14C, 14N and 14O
- **Isotone:** same neutron number (N) but different number of protons; Examples: 13B, 14C, 15N and 16O
- **Isomers (same *N* and *Z*)**: Nuclides can sometimes exist in different long-lived excited states. Isomers transition to other (more stable) nuclear configurations at different rates (half-lives). They also emit different quantities and types of ionizing radiation. Examples: Tellurium: ^{99m}Te is an isomer of ^{99}Te (superscript *m* stands for *metastable*); Silver: ^{108m}Ag is an isomer of ^{108}Ag

Ionization



Hydrogen (H): $^3_1\text{H}\rightarrow{}^3\text{H}\rightarrow\text{T}$

(also called ***tritium***, ***T***)

Mass number (A): **3**
Atomic number (Z): **1**
Neutron number (N): **2**

- The **mass of an atom** can be estimated by summing the masses of the electrons, protons and neutrons forming that atom
- The ***atomic mass unit*** (***u***) is define so that the mass of one 12C atom is exactly 12 u.
- The ***atomic weight*** of an atom is the ratio of the atom's mass to 1/12th the mass of a neutral atom of 12C in its ground state; naturally occurring elements are often a mixture of several isotopes. Atomic weight of an *element* is the sum of the atomic weight of each isotope times the ***isotopic abundance***
- **Avogadro's constant** is the number of atoms in 12 grams of 12C; Na = 6.02214150 x 10^{23} atoms/mol
- **Atom densities** (atoms cm^{-3}) on order of **10^{21} cm^{-3} to 10^{23} cm^{-3}** for solids and liquids; 1000X lower for gases.

$$A=\sum_i(\gamma_i/100)A_i$$

▹ γi is the % isotopic abundance

▹ Ai is the atomic weight

How many atoms in the human body? Water is the largest constituent of the human body, and the molecular weight of water is 18 u (18 g/mol). Suppose the human body weights 100 kg. Then $\frac{18\text{ g/mol}}{6.022\times10^{23}\text{atom/mol}}=2.989\times10^{-23}\text{g}/H_2O$

$$100\ \frac{\text{kg}}{\text{human body}}\cdot\frac{1000\text{ g}}{1\text{ kg}}\cdot\frac{\text{atom }H_2O}{2.989\times10^{-23}\text{g}}=3.3456\times10^{27}\ \frac{\text{atoms of }H_2O}{\text{human body}}$$

$$1\frac{\text{g}}{\text{cm}^3}\cdot\frac{H_2O}{2.989\times10^{-23}\text{g}}=3.35\times10^{22}\ \frac{H_2O}{\text{cm}^3}$$

Rest Mass & Energies

m_e = 0.51099890 MeV
m_p = 938.272 MeV
m_n = 939.56533 MeV
1 u = 931.494013 MeV

Fundamental Physical Constants — Frequently used constants

Quantity	Symbol	Value	Unit	Relative std. uncert. u_r
speed of light in vacuum	c, c_0	299 792 458	m s^{-1}	(exact)
magnetic constant	μ_0	$4\pi\times10^{-7}$	N A^{-2}	
		$=12.566\,370\,614\ldots\times10^{-7}$	N A^{-2}	(exact)
electric constant $1/\mu_0c^2$	ϵ_0	$8.854\,187\,817\ldots\times10^{-12}$	F m^{-1}	(exact)
Newtonian constant of gravitation	G	$6.6742(10)\times10^{-11}$	m^3 kg^{-1} s^{-2}	1.5×10^{-4}
Planck constant	h	$6.626\,0693(11)\times10^{-34}$	J s	1.7×10^{-7}
$h/2\pi$	$\hbar$	$1.054\,571\,68(18)\times10^{-34}$	J s	1.7×10^{-7}
elementary charge	e	$1.602\,176\,53(14)\times10^{-19}$	C	8.5×10^{-8}
magnetic flux quantum $h/2e$	Φ_0	$2.067\,833\,72(18)\times10^{-15}$	Wb	8.5×10^{-8}
conductance quantum $2e^2/h$	G_0	$7.748\,091\,733(26)\times10^{-5}$	S	3.3×10^{-9}
electron mass	m_e	$9.109\,3826(16)\times10^{-31}$	kg	1.7×10^{-7}
proton mass	m_p	$1.672\,621\,71(29)\times10^{-27}$	kg	1.7×10^{-7}
proton-electron mass ratio	m_p/m_e	$1836.152\,672\,61(85)$		4.6×10^{-10}
fine-structure constant $e^2/4\pi\epsilon_0\hbar c$	α	$7.297\,352\,568(24)\times10^{-3}$		3.3×10^{-9}
inverse fine-structure constant	α^{-1}	$137.035\,999\,11(46)$		3.3×10^{-9}
Rydberg constant $\alpha^2m_ec/2h$	R_∞	$10\,973\,731.568\,525(73)$	m^{-1}	6.6×10^{-12}
Avogadro constant	N_A, L	$6.022\,1415(10)\times10^{23}$	mol^{-1}	1.7×10^{-7}
Faraday constant N_Ae	F	$96\,485.3383(83)$	C mol^{-1}	8.6×10^{-8}
molar gas constant	R	$8.314\,472(15)$	J mol^{-1} K^{-1}	1.7×10^{-6}
Boltzmann constant R/N_A	k	$1.380\,6505(24)\times10^{-23}$	J K^{-1}	1.8×10^{-6}
Stefan-Boltzmann constant $(\pi^2/60)k^4/\hbar^3c^2$	σ	$5.670\,400(40)\times10^{-8}$	W m^{-2} K^{-4}	7.0×10^{-6}
Non-SI units accepted for use with the SI				
electron volt: (e/C) J	eV	$1.602\,176\,53(14)\times10^{-19}$	J	8.5×10^{-8}
(unified) atomic mass unit $1\text{ u}=m_u=\frac{1}{12}m(^{12}\text{C})=10^{-3}\text{ kg mol}^{-1}/N_A$	u	$1.660\,538\,86(28)\times10^{-27}$	kg	1.7×10^{-7}

$$Energy\ =\ E=h\nu\ =\ \frac{hc}{\lambda}=1.6\times10^{-19}\text{ J}=1\text{ eV (e}^-\text{ accelerated thru 1 V)}$$

$$Activity=A=3.7\times10^{10}\text{ Bq}=1\text{ Ci (1 g of }^{226}\text{Ra); 1 Bq}=1\text{ dps}$$

$$Proton=p={}^1_{+1}H=1.67262158\text{ x }10^{-27}\text{kg}=938.271998\text{ MeV}$$

$$Neutron=n={}^1_0n=1.67492716\text{ x }10^{-27}\text{kg}=939.565330\text{ MeV}$$

$$Electron=e^-={}^0_{-1}e=9.10938188\text{ x }10^{-31}\text{ kg}=0.510998902\text{ MeV}$$

$$1\ amu=A=1.66053873\text{ x }10^{-27}\text{ kg}=931.494013\text{ MeV}$$

$Alpha=\alpha={}^4_2He$ (α decay from low n-to-p ratio; ${}^A_Z\text{X}\rightarrow{}^{A-4}_{Z-2}\text{Y}+\alpha$)

$Beta=\beta^-={}^0_{-1}e$ (emission from high n-to-p ratio; $n\rightarrow p+\beta^-+\bar{\nu}_e$)

$Positron=\beta^+={}^0_{+1}e$ (emission from low n-to-p ratio if α not possible)

Electron Capture = Emission from low n-to-p ratio if α and β^+ not possible

Gamma Ray Emission = γ = *photons* (nuclei in excited state after transformation; ${}^A_Z\text{X}^*\rightarrow{}^A_Z\text{X}+\gamma$)

Photoelectric Effect = Vacancy in lower shell → characteristic x-rays/auger electrons emitted

▹ Photon interacts with entire atom → ejects photoelectron from K shell; $E_e=E_\gamma$ - $E_{binding}$

Compton Effect = Vacancy in lower shell → characteristic x-rays/auger electrons emitted

▹ Incident photon scatters off atomic electron; altered wavelength

Pair Production = Incident photon completely absorbed; replaced with $\beta^+:\beta^-$

▹ $E_\gamma\geq1.022$ MeV; $E_e=E_p=(E_\gamma-1.022)/2$

$$Couloum\ Barrier=\text{E}_{\text{CB}}=1.44\frac{\text{Z}_\text{P}\text{Z}_\text{T}}{\text{R}_\text{P}+\text{R}_\text{T}}MeV,\ R=1.4A^{1/3}$$

Size and density are related

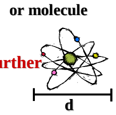
Atom or molecular density (Eq. 1.4): $N=\frac{\rho}{\text{A}}N_a$ Atom or molecules cm^{-3}

where N_a is Avogadro's constant, ρ is density (*g* cm^{-3}) and A is the atomic or molecular mass (g) of the substance. For *solids and liquids*, the effective volume and size of an atom or molecule is $\frac{1}{N}=\frac{1}{\frac{\rho}{\text{A}}N_a}=\frac{A}{\rho N_a}$ **cm³ per atom or molecule**

Effective diameter of atom or molecule

$$d=\left(\frac{1}{N}\right)^{1/3}=\left(\frac{A}{\rho N_a}\right)^{1/3}$$

Electron unlikely to be found further away from nucleus than *d*/2



Size of Nucleus (Section 1.2.8)

- The size of nucleus can be approximated by a sphere of radius where $R_0\cong1.25\times10^{-13}$ cm and A is atomic weight

$$R=R_0\text{A}^{1/3}$$

Lorentz Factor

$$\frac{1}{\gamma^2}=1-\beta^2 \quad \text{or} \quad \beta^2=1-\frac{1}{\gamma^2}$$

$$\gamma \equiv \frac{1}{\sqrt{1-(v/c)^2}}=\frac{1}{\sqrt{1-\beta^2}}$$

$$\gamma=\frac{T}{m_0c^2}+1=\frac{T+m_0c^2}{m_0c^2}$$

Compute γ to save time!

β = (v/c)	Lorentz factor (γ)
0.0001	1.000000005
0.001	1.000000500
0.01	1.000050004
0.1	1.005037815
0.2	1.020620726
0.3	1.048284837
0.4	1.091089451
0.5	1.154700538
0.6	1.250000000
0.7	1.400280084
0.8	1.666666667
0.9	2.294157339
0.99	7.088812050
0.999	22.366272042
0.9999	70.712445952

Mass

$$m=\frac{m_0}{\sqrt{1-(v/c)^2}}=\frac{m_0}{\sqrt{1-\beta^2}}=m_0\gamma$$

Velocity

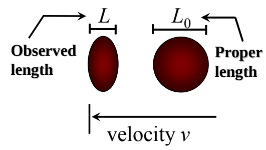
$$v=c\sqrt{1-\frac{1}{\gamma^2}}$$

Time Dilation

$$t=\frac{t_0}{\sqrt{1-(v/c)^2}}=\frac{t_0}{\sqrt{1-\beta^2}}=\gamma t_0$$

Lorentz Contraction

$$L=L_0\sqrt{1-\beta^2}=L_0/\gamma$$



Momentum

$$p=\sqrt{2mT}=\sqrt{\text{kg}\cdot\text{J}\frac{1\ \text{kg-m}^2/\text{s}^2}{1\ \text{J}}}=\sqrt{\text{kg}^2\text{m}^2/\text{s}^2}=\text{kg-m/s}$$

$$p=mv=(\gamma m_0)v=\text{kg}\cdot\text{m/s}$$

$$p=hv/c=h/\lambda$$

photon momentum

Classical! *Relativistic!*

Kinetic Energy

$$T=\frac{1}{2}mv^2=\frac{1}{2}(mv)\cdot(mv)\frac{1}{m}=\frac{p^2}{2m}$$

$$T=E-m_0c^2=\frac{m_0c^2}{\sqrt{1-\beta^2}}-m_0c^2$$

$$=m_0c^2\left[\frac{1}{\sqrt{1-\beta^2}}-1\right]$$

$$=(\gamma-1)m_0c^2$$

Total Energy

$$E=mc^2=\gamma m_0c^2$$

$$E(v)=\frac{m_0c^2}{\sqrt{1-(v/c)^2}}$$

$$E=\sqrt{(m_0c^2)^2+(pc)^2}=\sqrt{(\text{MeV})^2+(\frac{\text{MeV}}{\text{m/s}}\cdot\text{m/s})^2}=\text{MeV}$$

- UV radiation (**photon energy**): What is the kinetic energy of UV radiation with a wavelength of 290 nm?

$$v=\frac{c}{\lambda}=\frac{3\times10^8\,\text{m/s}}{290\ \text{nm}}\cdot\frac{10^9\,\text{nm}}{\text{m}}=1.0345\times10^{15}\text{s}^{-1}=1.0345\times10^{15}\,\text{Hz}$$

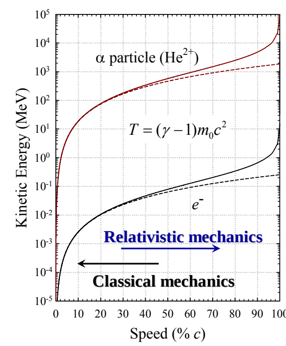
$$E=h\cdot v=6.626\times10^{-34}\,\text{J-s}\cdot(1.0345\times10^{15}\,\text{Hz})\cdot\frac{1\ \text{s}^{-1}}{1\ \text{Hz}}\cdot\frac{\text{eV}}{1.6022\times10^{-19}\ \text{J}}$$

$$=4.278\ \text{eV}$$

For comparison, 3.89 eV (Cs) to 24.59 eV (He) is the minimum amount of energy required to liberate a bound electron (i.e., produce ionization**)**

UV radiation with a wavelength shorter than about 124 nm (*E* = 10 eV) is often considered ionizing.

Kinetic energy as a function of speed



- Heavy particles must acquire more kinetic energy than light particles to become relativistic.
- For most application, particles traveling at speeds less than 0.001 to 0.01 *c* may be considered non-relativistic.
  - If an electron is not relativistic, heavier particles will not be relativistic

1 MeV = 1.602176462 × 10^{-13} J

Uncertainty position

$$\Delta x\Delta p \geq \frac{h}{2\pi}$$

← Planck's constant

Uncertainty momentum

Uncertainty in the location of an electron

$$p=mv=9.1\times10^{-31}\ \text{kg}\times300\ \text{m/s}=2.7\times10^{-28}\ \text{kg m/s}$$

$$\Delta p=m\Delta v=(0.01\%)\cdot2.7\times10^{-28}\ \text{kg m/s}$$

$$=2.7\times10^{-32}\ \text{kg m/s}$$

$$\Delta x\geq\frac{h}{2\pi\Delta p}=\frac{6.626\times10^{-34}\ \text{J-s}}{2\pi\cdot2.7\times10^{-32}\ \text{kg m/s}}$$

$$=0.00390\ \text{m}\ (=0.39\ \text{cm})$$

Conclusion: Location of e^- is highly uncertain

Why are atom's so big?

$$\frac{h}{2\pi\Delta x m_0}=\frac{6.626\times10^{-34}\,\text{J-s}}{2\pi\cdot0.0528\ \text{nm}}\cdot\frac{10^9\,\text{nm}}{\text{m}}\cdot\frac{\text{kg-m}^2/\text{s}^2}{1\ \text{J}}\cdot\frac{1\ \text{electron}}{9.11\times10^{-31}\text{kg}}$$

$$=2.1924\times10^6\,\text{m/s}$$

$$\Delta p=\gamma v\geq\frac{h}{2\pi\Delta x m_0}$$

Homework problem 2.14 and 2.15 relate to neutrons and electrons "trapped" in the *nucleus* (~ 10^{-15} m) instead of inside the *atom* (10^{-10} m)

$$\gamma v\geq2.1924\times10^6\,\text{m/s}\rightarrow v\geq2.1924\times10^6\,\text{m/s}$$

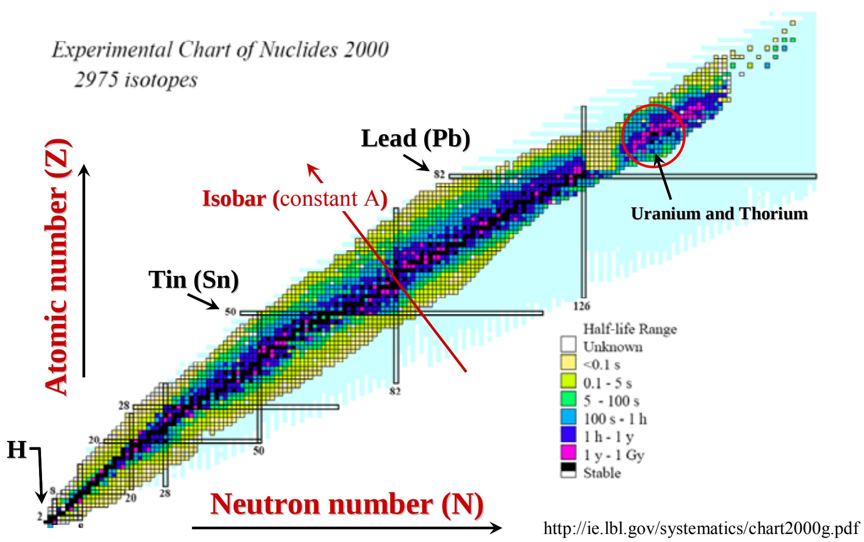
0.7308% of *c* (non-relatistic, so γ ~ 1)

$$T=\frac{1}{2}mv^2=\frac{1}{2}\cdot9.11\times10^{-31}\text{kg}\cdot(2.1924\times10^6\,\text{m/s})$$

$$=13.67\ \text{eV}$$

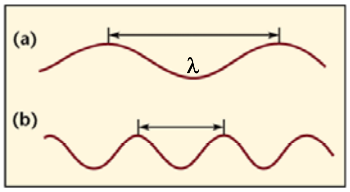
← **Kinetic energy (speed) of electron keeps it from spiraling into the nucleus**

Chart of the nuclides



http://ie.lbl.gov/systematics/chart2000g.pdf

Electromagnetic waves

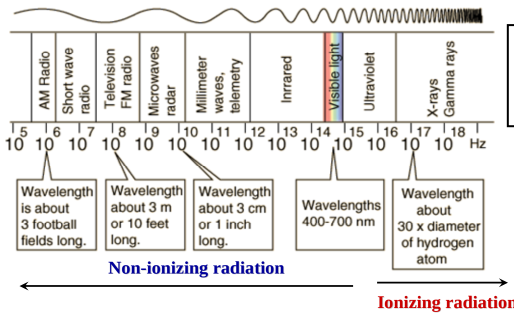


Speed of light (c) = wavelength (λ) × frequency (ν)

Energy (E) = Planck's constant (h) × ν

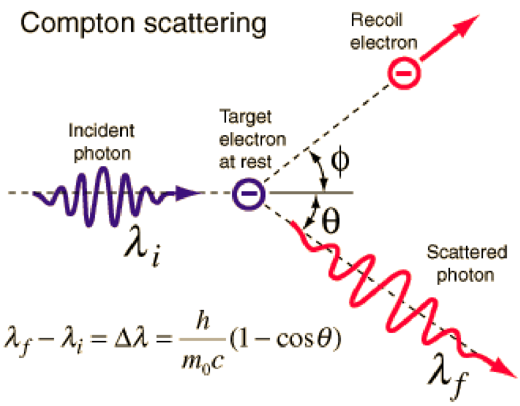
6.626×10^{-34} J-s

Electromagnetic Spectrum



- **X-rays**: Energy ~ 100 eV to a few tens or hundreds of keV; Wavelength: ~ 0.01 to 10 nm
- **Gamma-rays**: Energy: >

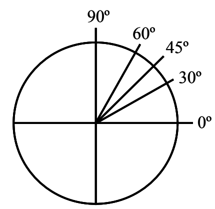
Compton scattering



$$\lambda_f - \lambda_i = \Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

Properties of cosine

$$\lambda' = \lambda + \frac{h}{m_e c}(1-\cos\theta_s)$$



$$\cos 0^\circ = 1$$
$$\cos 30^\circ = \frac{\sqrt{3}}{2} = 0.8660$$
$$\cos 45^\circ = \frac{\sqrt{2}}{2} = 0.7071$$
$$\cos 60^\circ = \frac{1}{2} = 0.5000$$
$$\cos 90^\circ = 0$$

$$\{\text{angle in degree}\} \cdot \frac{\pi}{180} = \{\text{angle in radian}\}$$

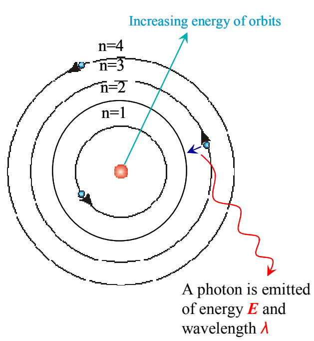
$$E' = \frac{E}{1 + E(1-\cos\theta_s)/m_e c^2}$$

$$\lambda' = \lambda + \frac{hc}{m_e c^2}(1-\cos\theta_s) = \lambda + \frac{4.1356673\times10^{-15}\ \text{eV-s}\cdot(3\times10^8\ \text{m/s})}{511\ \text{MeV}\cdot10^6\ \text{eV/MeV}}(1-\cos\theta_s)$$
$$= \lambda + (2.4279847\times10^{-15}\ \text{m})\cdot(1-\cos\theta_s)\cdot\{\text{unit conversion factor}\}$$

Development of the Modern Atomic Model

$$\frac{1}{\lambda_{n\to n_0}} = \frac{m_e Z^2 e^4}{8\varepsilon_0^2 c h^3}\left[\frac{1}{n_0^2} - \frac{1}{n^2}\right] = R_\infty\left[\frac{1}{n_0^2} - \frac{1}{n^2}\right], \qquad n > n_0$$

Bohr model revisited:



Hydrogen wavelength:

$$\frac{1}{\lambda} = R_H\left[\frac{1}{n_0^2} - \frac{1}{n^2}\right]$$

$R_H = 1.0967758 \times 10^7\ \text{m}^{-1}$

From particle duality: $E = h\nu$

$c = \lambda\nu$

$E = \frac{hc}{\lambda}$

R_H = *Rydberg constant*

h = *Planck's constant*

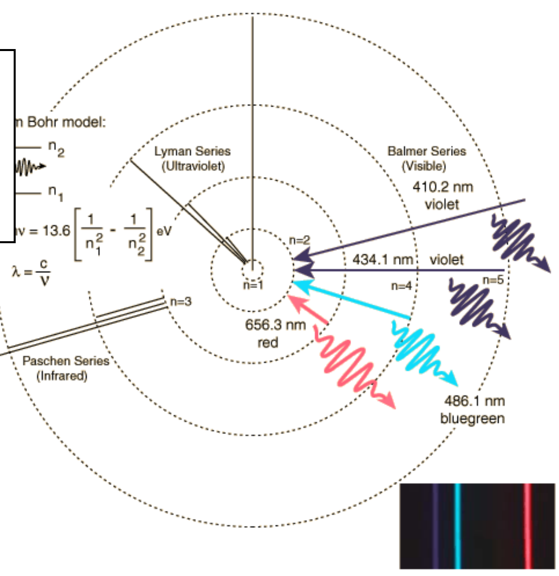
c = *Speed of light in vacuum*

$$\frac{1}{\lambda} = R_H\left[\frac{1}{n_0^2} - \frac{1}{n^2}\right] \qquad E_n = -\frac{m_e\left(Ze^2\right)^2}{8\varepsilon_0^2 n^2 h^2}, \qquad n = \sqrt{-\frac{m_e\left(Ze^2\right)^2}{E_n 8\varepsilon_0^2 h^2}}$$

$$R_H = 1.0967758 \times 10^7\ m^{-1}$$

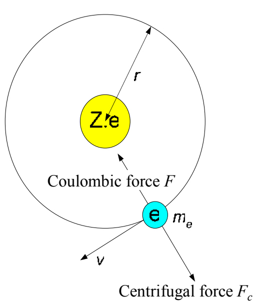
$$E_{n\to m} = \frac{m_e Z^2 e^4}{8\varepsilon_0^2 h^2}\left[\frac{1}{m^2} - \frac{1}{n^2}\right] = 13.6056923\ \text{eV}\cdot\left[\frac{1}{m^2} - \frac{1}{n^2}\right]Z^2$$

n=jump



Development of the Modern Atomic Model

Bohr model revisited:



Centrifugal force: $F_c = \frac{m_e v^2}{r}$

Coulombic force: $F = \frac{(Z\cdot e)\cdot e}{4\pi\varepsilon_0 r^2}$

For electron to remain in orbit: $\frac{m_e v^2}{r} = \frac{(Z\cdot e)\cdot e}{4\pi\varepsilon_0 r^2}$ [1]

Orbit defined from angular momentum L:

$L \equiv m_e v r = n\frac{h}{2\pi}$ [2], $n = 1,2,3,\ldots$

Simultaneously solve [1] and [2] to obtain:

$$v_n = \frac{Ze^2}{2\varepsilon_0 n h}, \quad r_n = \frac{n^2 h^2 \varepsilon_0}{\pi m_e Z e^2}, \quad n = 1,2,3,\ldots$$

ε_0 = *Permittivity of free space*